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ISBN 978-3-031-66476-2 ISBN 978-3-031-66477-9 (eBook)
https://doi.org/10.1007/978-3-031-66477-9

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Introduction

In the ever-evolving landscape of technology, quantum computing stands as one of the most revolutionary frontiers, promising to reshape the way we approach complex computational problems. At the heart of this groundbreaking field lies the concept of the qubit, the quantum analog of the classical bit. Unlike classical bits, which are confined to binary states of 0 and 1, qubits exploit the principles of quantum mechanics to exist in superpositions of states, enabling a new paradigm of information processing.

This book delves into the fascinating world of physical qubits—the foundational building blocks of quantum computers. While the theoretical framework of quantum computing has been well-established, the practical realization of qubits presents a multitude of challenges and opportunities. Various physical systems have been explored to implement qubits, each with its unique advantages, limitations, and technological requirements.

Quantum mechanics, with its counterintuitive and often bewildering principles, provides the underlying rules that govern qubit behavior. Superposition, entanglement, and quantum interference are not just theoretical curiosities, they are the phenomena that grant quantum computers their extraordinary power. However, harnessing these phenomena in a stable, scalable, and error-resistant manner is a formidable task.

The quest to create reliable physical qubits has led researchers down diverse paths, exploring a range of materials and techniques. From trapped ions and superconducting circuits to topological qubits and quantum dots, each approach offers unique insights and practical hurdles. Understanding the intricacies of these physical systems is essential for anyone aspiring to contribute to the field of quantum computing.

This book does assume some basic knowledge of quantum computing. For those readers who lack that knowledge, appendices are provided. Appendix 1 covers the basics of quantum physics and quantum computing. Appendix 2 covers essential math needed to understand quantum computing. Many readers won't need the material in these appendices, thus they are provided as appendices rather than chapters.

Contents

About the Author

Chuck Easttom is the author of 42 books, including several on computer security, forensics, and cryptography. He is also an inventor with 26 patents and the author of over 70 research papers. He holds a Ph.D. in computer science, a Ph.D. in Nanotechnology, and a Doctor of Science in Cybersecurity, and four master's degrees (one in applied computer science, one in education, one in Strategic and Defense studies, and one in systems engineering). He is a senior member of both the IEEE and the ACM. He is also a Distinguished Speaker of the ACM and a Distinguished Visitor of the IEEE. Dr. Easttom is currently an adjunct professor for Georgetown University and for Vanderbilt University. At Vanderbilt Dr. Easttom teaches a graduate course in quantum computing.

Chapter 1
Trapped Ion Quantum Computing

Introduction

There are numerous approaches to the physical implementation of qubits. Each of these approaches has advantages and disadvantages. In trapped ion quantum computing, qubits are represented by ions. These ions are trapped and isolated in a vacuum using electromagnetic fields. The ions are typically trapped using a device known as a Paul trap or a Penning trap. These traps use a combination of electric or magnetic fields to hold the ions in place. The ions are cooled down to near absolute zero temperatures to minimize their movement and interaction with the environment, which helps in maintaining their quantum state. The internal states of the ions (like electron energy levels or nuclear spin states) are used to represent the 0 and 1 states of a qubit. These states can be precisely controlled using lasers or microwave radiation. Lasers can be applied to the trapped ion to induce coupling between the qubit states or to induce entanglement.

Entanglement is a key resource in quantum computing that allows qubits to be correlated in ways that are impossible in classical computing. Ion based qubits are relatively easy to entangle. The final state of the ions is measured using a process called quantum state readout. This typically involves focusing a laser on the ion and detecting the light it scatters, which reveals the state of the qubit. Ion based qubits tend to experience long coherence times and high fidelity. There have been issues with scaling up to large numbers of qubits and the speed is often slower than other options such as superconducting qubits.

Decoherence can be more formally understood in the context of DiVincenzo's criteria. For those readers who might not be familiar with DiVincenzo's criterial, these are the criteria that are necessary for any usable quantum computer:

A scalable physical system with well-characterized qubit.
The ability to initialize the state of the qubits to a simple fiducial state.
Long relevant decoherence times.

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_1

A "universal" set of quantum gates.
A qubit-specific measurement capability.

There are two other criteria necessary for quantum communication, but these five are sufficient for quantum computation.

Trapping Ions

An ion trap is a device that utilizes electric and/or magnetic fields to capture an ion and isolate it from the environment. While the focus in this chapter will be on ion traps used in quantum computing, the need to trap ions predates quantum computing and has been used in other areas of physics. Frans Micheal Penning noted that electrons released by a cathode of an ionization vacuum gage follow a particular path to an anode, when in the presence of a magnetic field of sufficient strength. This led to work confining ions. Ion traps were used in the earliest television receivers and have been used in very accurate atomic clocks.

Trapping ions is a bit more challenging than one might first suspect. One reason for these challenges is Earnshaw's theorem, this theorem essentially states that a charged particle cannot be held in a stable equilibrium by electrostatic forces alone. For those readers who wish a more rigorous mathematical description, let us put this more formally: The Laplace has no solution with local minima or maxima in free space. There are saddle points, which is a point in the graph of a function where the derivatives (i.e., the slopes) in orthogonal directions are all zero but is not a local extrema of the function. What all that means is that using electrostatic forces alone, there cannot be a stable equilibrium. Put a bit more simply, the saddle point is the point of minimized energy magnitude for those ions that are trapped in the potential field. This theorem was proven by Samuel Earnshaw in 1842.

What Earnshaw's theorem means for quantum computing is that one cannot trap charged particles in three-dimensional space using only electrostatic forces. There must be something more to maintain the charged ions in a given location. What is normally used is an oscillating radio frequency. This forms a potential that has the shape of a saddle which is spinning at the frequency of the oscillating RF.

There are four basic methods of trapping ions. These are Paul trap, Penning trap, Kingdon trap, and magnetic bottle. However trapped ion quantum computing typically relies on either a Paul trap or a Penning trap. Thus, the Paul and Penning trap will receive the most discussion in this chapter, but the other two will be briefly described.

The concept of an ion trap is to create a potential the confines the ion to a specific physical region. Positively charged ions will move away from regions of high potential to regions of low potential. One way to visualize this is shown in Fig. 1.1.

Unfortunately, the field described in Fig. 1.1 simply cannot be created using only static electric fields. Static electric fields create a saddle shaped potential that traps the ion in one direction, but not the other direction. This is shown in Fig. 1.2.

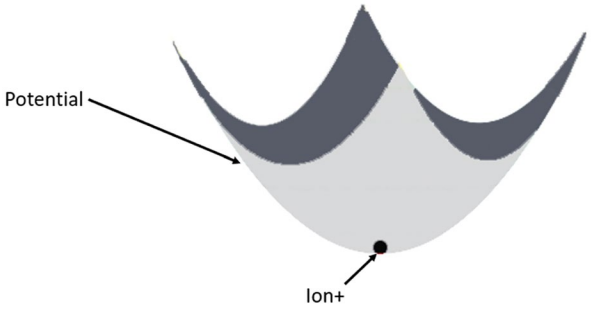
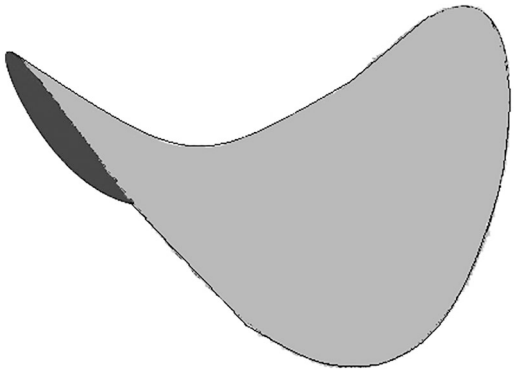


Fig. 1.1 Ion trap

Fig. 1.2 Saddle shaped potential



However, this saddle shaped potential only traps the ion in one direction. This means the ion will not remain trapped for very long. The answer is to rotate the potentials. If the saddle shaped potential rotates at the right frequency it will prevent the ion from escaping. A relatively straight forward equation describes the electric potential that is generated:

$$\Phi = \frac{1}{2}\left(u_X X^2 + u_y Y^2 + u_z z^2\right) + \frac{1}{\alpha}\left(v_X X^2 + v_y y^2 + v_z z^2\right)\cos\left(\omega t + \phi\right) \qquad (1.1)$$

In Eq. 1.1, the parameters u_i and v_i are adjusted to the mass and charge of the ion, and the ω (lower case omega) is adjusted to the potential's angular frequency. Those parameters are carefully tuned thus trapping the ion.

However, for quantum computing, trapping the ion would only be the first step. One then needs a mechanism to interact with the trapped ion. This includes applying gates and performing measurements. In trapped ion quantum computing, lasers are frequently used for this purpose. Lasers can be used to alter the state of electrons. If lower energy levels are not occupied, then the higher energy levels are unstable and the electrons at those levels will minimize their energy by moving to a lower level. Moving from a higher energy level to a lower, causes the emission of energy,

specifically a photon with energy equal to the energy lost by the electron. That energy is proportional to the frequency of the photon. When an electron is in a ground state, then a laser can be applied with the laser's frequency set to the difference in energy level between the ground state and an excited state and cause the electron to move to that higher state.

Trapped Ion Hardware

As was mentioned in the introduction, trapped ion quantum computing relies on either a Paul trap or a Penning trap. A Paul trap is named after Nobel Laureate Wolfgang Paul (note, this is not a typo, this physicist is different than Wolfgang Pauli, another Nobel Laureate physicist).

Paul Ion Trap

A Paul trap is named after Nobel Laureate Wolfgang Paul (note, this is not a typo, this physicist is different than Wolfgang Pauli, another Nobel Laureate physicist). With a Paul trap, an oscillating electric field is applied, as required by Earnshaw's theorem. The key to a Paul trap working properly is having the proper field strength and oscillation frequency.

A Paul ion trap utilizes an oscillating quadrupole field in conjunction with a static electric field in order to trap ions. This allows the Paul trap to maintain the ions in a fixed location in three-dimensional space. The combination of an oscillating quadrupole field and a static electric field produces a three-dimensional quadrupole potential. A quadrupole field is a type of electromagnetic field characterized by a spatial arrangement of charges or magnetic moments that give rise to a specific distribution of electric or magnetic field lines. This configuration is more complex than the simpler dipole fields, which are created by two equal but opposite charges or magnetic poles. With an electric quadrupole, you would typically have two positive and two negative charges. The arrangement is such that the charges or poles are symmetrically spaced, and the net charge (or magnetic moment) of the quadrupole is zero. This means that, unlike a dipole, a quadrupole does not have a simple positive or negative end. The field produced by a quadrupole falls off more rapidly with distance than the field from a dipole. It should be noted that there are more complex geometries possible for an ion trap, but these are generally not used for quantum computing.

In a quadrupole trap, the electrodes that are diagonally opposite each other and an ac voltage is applied to them. This electric field forms a potential that has the shape of a saddle spinning, and it is spinning at the radio frequency.

A Paul ion trap utilizes an oscillating quadrupole field in conjunction with a static electric field in order to trap ions. By trapping it is meant to keep the ions in a

fixed location. The combination of an oscillating quadrupole field and a static electric field produces a three-dimensional quadrupole potential.

For those readers who are not well versed in electromagnetic physics, a quadrupole field refers to a type of electromagnetic field characterized by a spatial arrangement of charges or magnetic moments that gives rise to a particular distribution of electric or magnetic field lines. This configuration is more complex than the simpler dipole fields, which are created by two equal but opposite charges or magnetic poles. In a quadrupole field, there are two pairs of charges or poles. In the case of an electric quadrupole, you would typically have two positive and two negative charges. The arrangement is such that the charges or poles are symmetrically spaced, and the net charge (or magnetic moment) of the quadrupole is zero. This means that, unlike a dipole, a quadrupole does not have a simple positive or negative end. The field produced by a quadrupole falls off more rapidly with distance than the field from a dipole.

The electrodes that are diagonally opposite each other are connected and an ac voltage is applied to them. The electrical field produced by this current can be defined as shown in Eq. 1.1.

The basic physical structure of a quadrupole Paul ion trap consists of a ring electrode field that is held between two end-cap hyperbolic electrodes. Ions are trapped by applying an oscillating AC voltage to the ring electrode, which in turn causes the ring to produce a quadrupole electric potential that traps the ion. That basic structure is shown in Fig. 1.3.

Figure 1.3 simply represents the general overall structure of a Paul Ion Trap. This takes us back to the Paul ion trap. This trap uses an electric field oscillating at radio frequency. Assuming the frequency has the right oscillation frequency and field

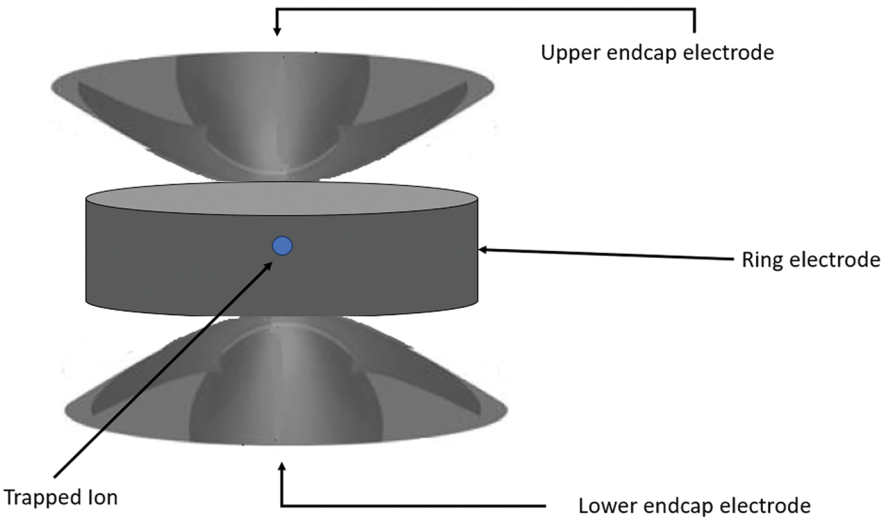


Fig. 1.3 Paul Ion Trap

strength, the charged particle is trapped at the aforementioned saddle point. For those readers who wish a bit more detail on how Paul ion traps and saddle points work, there is the Mathieu function. The motion of the ion in the saddle point is described by Mathieu functions. These functions are solutions to Mathieu's differential equations, first introduced by Emile Mathieu. The differential equation is shown here:

$$\frac{d^2y}{dx^2} + \left(a - 2q\cos 2x\right)y = 0 \tag{1.2}$$

In Eq. 1.2 a and q are parameters specific to the particular application. For more details on the Mathieu differential equations, the interested reader can consult the following sources:

http://pi.math.cornell.edu/~rand/randpdf/AMR.pdf
https://cds.cern.ch/record/828858/files/978-3-540-26576-4_BookBackMatter.pdf

Penning Trap

A Penning trap stores charged particles using a quadrupole electric field and a homogeneous magnetic field. The Penning trap was developed by Frans Michel Penning and Hans Dehmelt. Typically, a Penning trap has two endcap electrodes and one ring electrode. By applying a positive or negative potential to the ring electrode, relative to the endcaps, an electric field is generated. A uniform magnetic field is applied along the axis of the trap, which is perpendicular to the plane of the ring electrode. The trap is placed in a vacuum environment in order to minimize any collisions between trapped particles and residual gas molecules.

The magnetic field confines the particles radially, and the quadrupole electric field confines the particles axially. Energy can be removed from ions in a penning trap using a number of techniques including laser cooling, buffer gas cooling, and resistive cooling. Buffer gas cooling relies on neutral gas molecules. Laser cooling utilizes lasers to reduce the energy of the ions. Resistive cooling actual resistors are used.

Penning traps are not widely used in quantum computing, rather Paul traps are most often used, at least as of this writing. However, Penning traps are used in mass spectrometry and other areas of physics.

Kingdon Trap

With the Kingdon ion trap, a cylindrical cathode wire is placed on the axis of a cylindrical anode shell with endcap electrodes at both ends. A voltage is applied resulting in a radial logarithmic potential between the electrodes. Essentially particles with a nonzero angular momentum and a charge that is opposite to the wire, will revolve around the wired and thus be trapped.

There are several variations of the Kingdon trop, such as the Knight trap. The Knight trap (also called an ideal Kingdon trap) is an improvement on the Kingdon trap.

In 2024, researchers made a substantial improvement in ion traps.[1] Paul ion traps are limited in size. The oscillating fields make it difficult to combine several traps. A team of researchers at ETH Zurich led by Jonathan Home has now demonstrated that ion traps suitable for use in quantum computers can also be built using static magnetic fields instead of oscillating fields. In those static traps with an additional magnetic field, called Penning traps, both arbitrary transport and the necessary operations for the future super-computers were realized.

The researchers also demonstrated that the qubit energy states of the trapped ion could also be controlled while maintaining quantum mechanical superpositions. Coherent control worked both with the electronic (internal) states of the ion and the (external) quantized oscillation states as well as for coupling the internal and external quantum states. This latter is a prerequisite for creating entangled states, which are important for quantum computers.

Magnetic Bottle

A magnetic model uses an axially non-uniform magnetic fields. The field regions are stronger at the ends than in the middle. A charged particle can be confined in the radial plane by the Lorentz force that the magnetic field exerts on a moving charge. The magnetic bottle typically consists of a combination of two magnetic fields: a toroidal (doughnut-shaped) field and a poloidal (looping around the torus) field. These fields create a complex, twisted path that charged particles follow, effectively trapping them within the device. Outside of quantum computing, magnetic bottles are used in fusion reactions.

[1] https://www.nature.com/articles/s41586-024-07111-x

Using Trapped Ions

Having the physical architecture for trapped ion quantum computing is only the first step. Next, the qubits must be able to be measured, and to have specific gates applied to them. This is the process of actually utilizing trapped ion qubits. Initializing trapped ion qubits can be done with a process known as optical pumping. The optical pumping process is used to raise electrons from a lower energy level to a higher one. In the case of trapped ion qubits, a laser couples the ion to an excited state which will eventually decay to a lower state that is not coupled to the laser. When the ion reaches that lower state, it will remain in that state. If for some reason the decay does not lead to a stable lower state, then the laser will be used to excite the ion again until it does decay into the lower state which does not interact with the laser.

Measurement

The measurement of the state of an ion qubit is relatively straight forward. A laser is applied to the ion that couples only one of the qubit states. Assuming that particular qubit state, then during the measurement process, the laser excites the ion, ultimately causing a photon to be released when the ion decays from the excited state. The laser continues to excite the ion and continues emitting photons. These photons are collected, usually by either a charged coupled device camera or a photomultiplier tube. If the ion instead collapses into the other qubit state, then it will not interact with the laser and therefore no photon gets emitted. The photons are counted thus determining the state of the ion qubit when it collapsed.

A charge coupled device is an integrated circuit that contains an array of coupled capacitors. Each capacitor can transfer its electric charge to a neighboring capacitor. You may have used devices that have charged-coupled capacitors in devices related to digital imaging. A photomultiplier tube is a device that converts photons into an electrical signal. There are several specific types of photomultiplier tubes. One common type is called a silicon photomultiplier. This is a solid-state device that is sensitive to a single photon and uses a single photon avalanche diode (SPAD). The SPADs usually vary between 10 and 100 micrometers and are often packed with 10,000 SPADs per millimeter. SPADs are sometimes referred to as Geiger-mode avalanche photodiodes (GM-APD or G-APD). Like many components used in quantum computing, SPADs are neither new nor specific to quantum computing.

Another type of photomultiplier is the photomultiplier tube, sometimes called PMTs. These are very sensitive light detectors able to detect light in the visible, near-infrared, and ultraviolet wavelengths. PMTs multiply the current of the incident light by as much as 10^8. Another type of photomultiplier is the electrostatic photomultiplier. These have been around since the 1930s and are frequently used in commercial applications.

Gate Operations

Measurement and gate operations are key to performing anything useful with quantum computers, regardless of the physical basis for the qubits. With ion-based qubits manipulating the frequency of the electromagnetic field for specific periods of time can cause rotation operators. Specifically, Rx(θ) and Ry(θ). Rotationally operators are based on the Bloch sphere representation of the qubit. These two rotational operators are arbitrary single qubit rotations. Combining such rotations with a CNOT gate it is possible to create any quantum gate. This leads logically to the CNOT gate in trapped ion quantum computing.

In trapped ion systems arbitrary single qubit rotations can be accomplished utilizing magnetic dipole transitions. Another mechanism for affecting arbitrary single qubit rotations is simulated Raman transitions.

Challenges

As you are undoubtedly aware, the single most pressing issue in quantum computing is that of decoherence. As you can probably surmise, different approaches to physical qubits have different susceptibilities to decoherence. With ion trap quantum computing, when an ion is being transported between regions in an interconnected trap, if it is subjected to a nonuniform magnetic field this can lead to decoherence. That decoherence is related to the Zeeman effect. The Zeeman effect was discovered in 1986 and is named after its discoverer, Nobel Laureate Pieter Zeeman. The Zeeman effect is splitting a spectral line into several components in the presence of a static magnetic field. A related effect is the Stark effect that splits a spectral line into several components in the presence of a magnetic field. One method to mitigate decoherence is to represent quantum states in a new basis that is called the decoherence free subspace (DFS).

Ytterbium 171

As you will see later in this chapter, many vendors use Ytterbium is fairly reactive. It reacts slowly with water and quickly with diluted acids. It also oxidizes in air, particularly at elevated temperatures. 171^{+} for the qubits in trapped ion quantum computing. You may be wondering why this ion and not others. There are certainly good reasons for this, but first it may be beneficial to at least some readers to have a brief discussion of Ytterbium itself, apart from its used as a trapped ion qubit.

Ytterbium is a chemical element with the symbol Yb and atomic number 70 and an atomic weight of 173.04 atomic mass units. It is one of the lanthanides, a group of rare earth elements, and has several notable characteristics. Ytterbium is a soft,

malleable, and ductile metal with a bright silvery luster when freshly cut, but it can oxidize in air and become dull in appearance. Ytterbium melts at approximately 824 °C and boils around 1196 °C. It has a density of about 6.90 g/cm^3 at room temperature, which is relatively high, but typical for lanthanides. Ytterbium is fairly reactive. It reacts slowly with water and quickly with diluted acids. It also oxidizes in air, particularly at elevated temperatures. The electron shell configuration of Ytterbium in its ground state is $[Xe].4f^{14}.6s^2$, this means it has the same configuration as Xeon, plus the additional shell information. If you prefer the full configuration written out it is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 4f^{14}\ 5s^2\ 5p^6\ 6s^2$. The shell configuration is shown in Fig. 1.4.

Ytterbium-171 has a nuclear spin of 1/2, which is advantageous for quantum computing. This spin property allows for the creation of hyperfine qubit states, where qubits can be represented by the different energy levels associated with the nuclear spin states. These states are less susceptible to decoherence caused by external magnetic fields, making them highly stable for quantum operations. The nuclear spin of ^171Yb provides inherent resistance to decoherence due to magnetic field fluctuations. This is crucial for maintaining the coherence of qubits over longer durations, which is essential for performing complex quantum algorithms and error correction routines.

Ytterbium ions have electron transitions that are suitable for manipulation using commercially available lasers. ^171Yb, in particular, can be manipulated using lasers in the visible and near-infrared spectrum. This is particularly useful for initializing, manipulating, and reading out the quantum state of the ions using optical methods. The energy levels in ^171Yb allow for the use of frequency and phase manipulation techniques to address individual ions selectively, even when they are closely spaced in a trap. This selectivity is crucial for scaling up the number of qubits in a quantum system, as it allows for individual control over each qubit

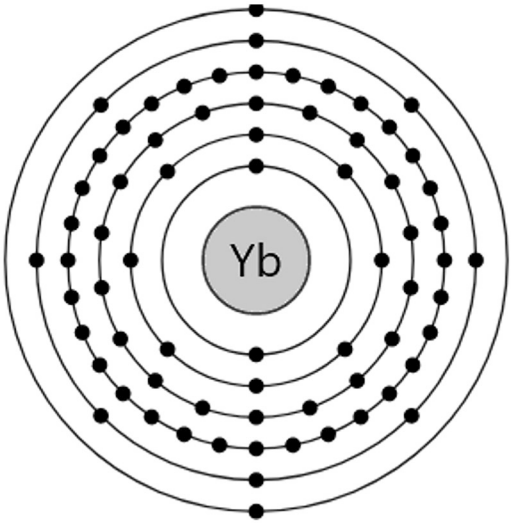


Fig. 1.4 Ytterbium electron configuration

without affecting its neighbors. The hyperfine qubits in ^171Yb ions exhibit long coherence times, which are critical for performing multiple successive quantum operations before quantum information is lost. This makes ytterbium ions well-suited for experiments requiring complex quantum manipulations over extended periods.

Perhaps more important than other factors, Ytterbium is a relatively abundant and stable element. Its isotopes, including ^171Yb, can be isolated and enriched without excessive cost or technical difficulty, making it a practical choice for experimental and commercial quantum computing setups.

Specific Implementations

Now that you have covered the essentials of trapped ion quantum computing, it is useful to examine specific implementations. In the following subsections we will examine architectures and processes used by particular vendors.

IonQ

The company IonQ is one of the leaders in trapped ion quantum computing. It therefore is logical to explore their approach. IonQ advertises that its qubits are completely identical with no variations. The qubits are then assembled into a chain using an ion trap chip. A chain of qubits is essentially a single core quantum processing unit (QPU). With multicore quantum processing units, IonQ works with the premise of Reconfigurable Multicore Quantum Architecture (RMQA).

The idea behind RMQA is to create a quantum computing architecture that is not only powerful but also flexible and adaptable. RMQA systems consist of several quantum cores that can operate independently or in conjunction with each other. This modularity enhances the flexibility and scalability of quantum systems. The interconnections between quantum cores can be reconfigured dynamically. This means that the system can adapt its architecture to optimize performance for specific tasks, such as quantum simulations, optimization problems, or cryptographic computations.

This adaptability comes from the reconfigurable nature of the architecture, allowing it to be optimized for different quantum algorithms or applications. For instance, one configuration of RMQA might be well-suited for simulating quantum chemistry, while another could be optimized for cryptography or optimization problems. The term “multicore” implies that the architecture involves multiple processing units or cores that work in parallel. In classical computing, this often means having multiple CPU cores on a single chip. In the context of quantum computing, multicore suggests the presence of multiple quantum processing units. Reconfigurability in computing means the ability to change the configuration or

functionality of a system on-the-fly. In the context of RMQA, reconfigurability implies that the quantum cores or components of the architecture can be dynamically adjusted or customized to suit the specific quantum algorithms or tasks being executed.

In 2023, IonQ announced its Forte computer. This device uses ytterbium ions and is software configurable. Acousto-optic deflectors direct laser beams at individual qubits in the chain of ions in order to apply quantum gates. The device supports up to 32 qubits. The system employs laser and microwave technology to manipulate and control the qubits with high precision, reducing error rates and enhancing computational accuracy. IonQ Forte can integrate with classical computing resources, enabling hybrid quantum-classical computing approaches. This integration allows for the optimization of workloads by leveraging the strengths of both quantum and classical processors.

Sandia Labs

Sandia Labs is known for a wide range of cutting-edge research. It should be no surprise that this lab is also working on quantum computing.[2] Their most well-known project is Quantum Scientific Computing Open User Testbed (QSCOUT).[3] This is a testbed to allow scientists to access trapped ion quantum computing for research. The first ion trap developed at QSCOUT was named Peregrine and was a linear ion trap. This linear architecture utilizes a chain of Ytterbium 171+ ions.

The next trap was a Y-shaped junction trap and is frequently used in error correction. As with other ion trap architectures, a laser is used to interact with the trapped ion qubits. QSCOUT allows Python programming with their system. Details of this architecture have been published by Sandia labs.[4] Sandia national labs has also patented the y-shaped junction.[5]

QSCOUT also supports a quantum gate assembly language named Jaqal (Just Another Quantum Assembly Language). One-way Jaqal is used is to create Gate Pulse Files (GPF). These files specify the laser pulses that will implement trapped ion gates. Jaqal also allows users to create composite gates using sub-circuit macros. The operations and gates supported by Jaqal are listed here:[6]

- Prepare_all Prepares all qubits in the quantum register in the $|0\rangle$ state in the z basis.

[2] https://www.sandia.gov/quantum/trapped-ions/

[3] https://www.sandia.gov/quantum/quantum-information-sciences/projects/qscout/

[4] https://arxiv.org/pdf/1105.1834.pdf

[5] https://patents.google.com/patent/US10984976B1/en

[6] https://www.sandia.gov/app/uploads/sites/174/2022/02/quantum_assembly_spec.pdf

- R <qubit> <axis angle> <rotation angle> Counter-clockwise rotation around an axis in the equatorial plane of the Bloch sphere defined by <axis-angle>, measured counter-clockwise from the x axis, by the angle defined by <rotation angle>.
- Rx <qubit> <rotation angle> Counter-clockwise rotation around the x axis, by the angle defined by <rotation angle>.
- Ry <qubit> <rotation angle> Counter-clockwise rotation around the y axis, by the angle defined by <rotation angle>.
- Rz <qubit> <angle> Counter-clockwise rotation around the z axis, by the angle defined by <rotation angle>.
- Px <qubit> Counter-clockwise rotation around the x axis, by π. (Pauli X gate).
- Py <qubit> Counter-clockwise rotation around the y axis, by π. (Pauli Y gate).
- Pz <qubit> Counter-clockwise rotation around the z axis, by π. (Pauli Z gate).
- Sx <qubit> Counter-clockwise rotation around the x axis, by π/2. ($\sqrt{X}$ gate).
- Sy <qubit> Counter-clockwise rotation around the y axis, by π/2. ($\sqrt{Y}$ gate).
- Sz <qubit> Counter-clockwise rotation around the z axis, by π/2. ($\sqrt{Z}$ gate).
- Sxd <qubit> Clockwise rotation around the x axis, by π/2. ($\sqrt{X}$† gate).
- Syd <qubit> Clockwise rotation around the y axis, by π/2. ($\sqrt{Y}$ † gate).
- Szd <qubit> Clockwise rotation around the z axis, by π/2. ($\sqrt{Z}$† gate).
- MS <qubit> <qubit> <axis angle> <rotation angle>.

More information on Jaqal can be found in published papers available online[7] and on GitHub.[8]

Quantinuum

Formerly known as Honeywell, Quantinuum has been making advances in ion trapped quantum computing.[9] Their system, named System Model H1, is their first-generation quantum computer that has 20 fully connected qubits, but can have a volume of 2^{20}. Quantinuum asserts that they have achieved 99.9979% fidelity with single-qubit gates and 99.914% fidelity with two-qubit gates. The System Model H1uses Ytterbium 171^{+} ions. All quantum gates are implemented using lasers. They offer a product specification sheet online.[10]

System model H2 is their most recent version, as of 2024. It has 32 fully connected qubits, but a volume of 2^{16}. The System Model H2 has achieved 99.997% fidelity with single-qubit gates, and 99.8% fidelity with two-qubit gates. The H2

[7] https://arxiv.org/pdf/2003.09382.pdf

[8] https://github.com/sandialabs/JaqalPaq

[9] https://www.quantinuum.com/hardware

[10] https://assets.website-files.com/62b9d45fb3f64842a96c9686/661fec8ee992a15703fc67b5_Quantinuum%20H1%20Product%20Data%20Sheet%20v6.1%2015April24.pdf

also uses Ytterbium 171^+ ions for the qubits, and lasers for gates. There is also on online product specification sheet for this product.[11]

Honeywell's quantum computers are used for a variety of applications, including optimization problems, material science, pharmaceuticals, and finance. Their high accuracy makes them suitable for tasks that require precise calculations and simulations. In addition to hardware, Honeywell provides software and tools to facilitate the use of their quantum computers. This includes quantum programming environments and algorithms tailored to exploit the capabilities of their quantum processors.

Eleqtron

Eleqtron is a German based company specializing in ion trapped quantum computing. The company Eleqtron describes its technology as Magnetic Gradient Induced Coupling (MAGIC) which the use to control ion qubits.[12] Right now, they only have a demonstration model and do not yet have a model available to the public. They stated that they will have a 60-qubit system available soon, they call MAGIC Ruby.

The company is involved in several significant projects, including the EPIQ project in partnership with the Jülich Supercomputing Centre. This project aims to develop a modular quantum supercomputer integrating both quantum and classical digital modules. By the end of 2024, Eleqtron plans to build a pilot system featuring up to 30 trapped ion qubits, which will be expanded to 60 qubits by 2026. This system will support applications in optimization tasks across various industries, including logistics, traffic optimization, and process engineering, as well as in scientific fields such as physics, chemistry, biology, and medicine.

Alpine

Alpine Quantum Computing[13] is another company focused on trapped ion quantum computing. This company has focused on what can be done with quantum computers. So far, they have implemented a scalable implementation of Shor's algorithm; encoded quantum error correction; and entangled 14 particles.[14] Their current systems are IBEX and MARMOT with a quantum volume of 128. Their PINE system

[11] https://assets.website-files.com/62b9d45fb3f64842a96c9686/661d8a0f003b8ebcf781904f_Quantinuum%20H2%20Product%20Data%20Sheet%20v1.3%2015April24.pdf

[12] https://eleqtron.com/

[13] https://www.aqt.eu/

[14] 106.

is a 20-qubit quantum computer rack that they assert is ready for integration in existing infrastructure.[15]

Alpine Quantum Computing's technology uses electric fields to trap single charged atoms (ions) inside vacuum chambers, with each ion representing a quantum bit (qubit). These qubits are manipulated and measured using precisely timed laser pulses. The company has achieved significant milestones, including realizing a scalable Shor algorithm and topologically encoded quantum error correction. One of AQT's key products is a modular ion-trap quantum computer with a capacity of more than 100 qubits. This system is designed to be highly energy-efficient, requiring less than 2 kW of power and capable of being powered from a standard wall socket. This quantum computer can be directly integrated into existing high-performance computing (HPC) infrastructures via a cloud connection, making it accessible for a wide range of applications in both research and industry.

Quantum Factory

Quantum Factory is a German company founded in 2019 that works on trapped ion quantum computing. Quantum Factory is currently in the phase of securing funding and establishing partnerships. They offer consulting services and are actively engaging with potential investors and government bodies to advance their technology.[16]

Research

There exists a rich body of research articles focused on trapped ion qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2023 article published by the American Physical Society,[17] titled "Scaling Up a Trapped-Ion Quantum Computer" reviews significant advancements made by Steven Moses and colleagues at Quantinuum, in trapped ion quantum computing. The focus is on the H2 discussed previously in this chapter. The Quantinuum System Model H2 represents a major step in trapped-ion quantum computing. It now incorporates 32 qubits with no increase in error rates compared to its predecessor, which had 20 qubits. The system utilizes a quantum charge-coupled device (QCCD) architecture. This innovative setup allows for the dynamic rearrangement of ion chains,

[15] https://www.aqt.eu/pine-system-20-qubit-control/

[16] https://quantum-factory.de/

[17] https://physics.aps.org/articles/v16/209

enhancing the system's scalability and flexibility by enabling ions to interact across different sections of the device.

A 2024 article in Physics World,[18] describes a new ion trapping approach developed by researchers at ETH Zurich that could significantly aid the scaling up of quantum computers. Unlike traditional methods that use oscillating electromagnetic fields to trap ions, the ETH Zurich team uses static magnetic and electric fields. This method allows for better control of the ions' quantum states and positions, which is crucial for scaling up the number of qubits in quantum computers. Currently, trapped-ion quantum processors use radio frequency (RF) fields to confine ions, suitable for systems up to about 30 qubits. However, scaling this method to larger systems is difficult due to technical challenges such as interference and heat generation within the chip's small space.

The method described in the article utilizes Penning traps, described earlier in this chapter, traditionally employed in precision spectroscopy and quantum simulations, which replace RF fields with strong static magnetic fields. This change addresses the heating issue and allows for a more flexible trap configuration. The Penning trap setup includes a microfabricated chip with multiple electrodes placed inside a superconducting magnet, with phase-locked laser beams directed to the ions through a system of mirrors. The setup, cryogenically cooled and operated in a vacuum, demonstrated the ability to maintain an ion's quantum state for days, surpassing the duration needed for quantum operations.

Conclusions

Trapped ion quantum computing provides a robust approach to physical qubits. Trapped ion qubits tend to have long coherence times. Ion trapped qubits do require precise control over the electric field parameters and can be sensitive to external disturbances such as magnetic fields or vacuum fluctuations. A number of companies have experienced substantial success with the trapped ion approach to physical qubits. There have also been substantial research gains related to trapped ion quantum computing.

[18] https://physicsworld.com/a/new-ion-trapping-approach-could-help-quantum-computers-scale-up/

Chapter 2
Superconducting Quantum Computing

Introduction

Superconducting quantum computing is a common approach to quantum computing, used by companies such as Google and IBM. Specifically Googles' Sycamore chip and IBM's Eagle chip are examples of superconducting quantum computers. Materials such as selenium or germanium, or defective materials such as diamonds, aluminum nitride or silicon carbide are often used. Applying microwaves and magnetic fields to these materials will allow them to exhibit superposition, entanglement, and other quantum properties. Superconducting quantum computers are often cooled in dilution refrigerators below 15 mK. IMEC, Rigetti, and Intel also utilize superconducting quantum computers.

As with all approaches to the physical implementation of quantum computers, superconducting quantum computers have both advantages and disadvantages. One advantage is that superconducting qubit system has high designability. Different types of superconducting qubits, such as charge qubits, flux qubits, and phase qubits, can be designed. And different parameters, such as the energy level of the qubit and the coupling strength, can also be adjusted by adjusting the capacitance, inductance, and Josephson energy.

Another advantage is superconducting qubits have high designability. The preparation of superconducting qubits is based on the existing semiconductor microfabrication process. High-quality devices can be prepared by leveraging advanced chip-making technologies, which is good for manufacturing and scalability. The circuit nature of the superconducting qubit system makes it relatively easy to couple multiple qubits together. In general, superconducting qubits can be coupled by capacitance or inductance.

Superconducting qubits are also relatively easy to control. The operation and measurement of superconducting qubits compatible with microwave control and

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_2

operability. Thus, commercial microwave devices and equipment can be used in superconducting quantum computing experiments.

Of course, superconducting qubits also have some disadvantages. The more prominent of these is that these physical qubits experience decoherence faster than some other physical qubits. Decoherence is a substantial obstacle to quantum computing. A second issue is that these types of physical qubits tend to require more error correction. We will discuss error correction methods in Chap. 12. To overcome decoherence, and to facilitate super conduction, superconducting quantum computing is typically done in extremely cold temperatures, sometimes lower than 100 milli-Kelvin.

Technical Details

Superconductors have a critical temperature at which resistivity falls to very close to zero and conductivity is substantially increased. Charge is carried by pairs of electrons rather than single electrons. These pairs are called "Cooper pairs." Cooper pairs were first described by physicist Leon Cooper in 1956 but are also known as Bardeen-Cooper-Shrieffer pairs. The latter name is due to the Bardeen-Cooper-Schrieffer theory explaining the activity of cooper pairs. The three shared the 1972 Nobel prize in physics for their work. Bardeen-Cooper-Schrieffer essentially explains the "why" for Cooper pairs. The formation of Cooper pairs leads to the creation of an energy gap in the superconductor's electronic structure. This gap, which forms at the Fermi surface (the surface in momentum space that separates occupied from unoccupied electron states at zero temperature), means that a particular amount of energy is required to break the pairs apart. At low temperatures, thermal energy is insufficient to break these pairs, allowing them to move without resistance, which is the essence of superconducting. Bardeen-Cooper-Schrieffer theory also provides an explanation for the Meissner effect, wherein a superconducting material will expel all magnetic fields as it transitions into the superconducting state. This effect is a direct consequence of the macroscopic quantum state and the rigidity of the phase of the wave function of the Cooper pairs.

With superconducting qubits, the ground and excited states of atoms are mapped to the 0 and 1 values, thus encoding information. Superconducting quantum computers utilize Josephson junctions. A Josephson junction is something that occurs when two superconductors are in close proximity. This produces a current, in fact a supercurrent, which continues flowing without any voltage applied. This leads to macroscopic quantum phenomenon.

For any qubit implementation the logical quantum states are mapped to different states of the physical system, this is often discrete energy levels. The phase qubit uses a Josephson junction and energy levels correspond to different quantum charge oscillation amplitudes across that Josephson junction. The charge and phase are analogous to the momentum and position. The phase qubit's Josephson junction has a Josephson to charging energy ratio on the order of 10^6.

The flux qubit also uses a Josephson junction but with a Josephson to charging energy ratio on the order of magnitude 10. The energy levels correspond to different integer numbers of the magnetic flux quanta trapped in the superconducting ring. The flux qubit is sometimes referred to as a persistent-current qubit.

The charge qubit has a Josephson to charging energy ratio on the order of magnitude of <1. Different energy levels correspond to an integer number of Cooper pairs on a superconducting island. A superconducting island is an area, usually small, with a controllable number of charge carriers. A Cooper pair is a pair of fermions that are bound together at low temperatures in a particular manner first described by Leon Cooper in 1965. The charge qubit is sometimes called a Cooper pair box.

Figure 2.1 shows a basic circuit diagram of a charged qubit. In Fig. 2.1, C is a capacitor the Josephson junction has an energy Ej and it is biased by voltage U.

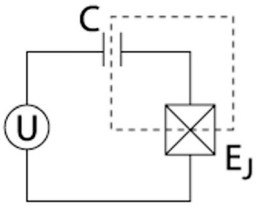
There are three primary supercomputing qubit types: phase, charge, and flux. However, there are numerous hybridizations of these three including the Fluxonium, transmon, and xmon. The phase qubit utilizes a superconductor-insulator-superconductor Josephson junction (SIS). SIS is also known as the superconducting tunnel junction (SJT). Whichever name is applied, the device consists of two superconductors that are separated by a very thin layer of insulating material.

Fluxonium qubits were first seen in 2009. Fluxonium qubits generally maintain coherence time longer than transmon qubits. Unlike other superconducting qubits, fluxonium qubits incorporate a large array of Josephson junctions. This array is in parallel with a single Josephson junction and a capacitance. The array of Josephson junctions creates a high inductive environment, which is central to the qubit's operation.

The fluxonium qubit has distinct energy levels, like other qubits. These energy levels can be precisely controlled by external magnetic flux. The energy level structure of a fluxonium qubit allows for transitions between states that are more resilient to certain types of noise, especially charge noise, compared to other superconducting qubits. The fluxonium qubit is tunable, meaning its energy levels can be adjusted by changing the external magnetic flux through the qubit. This tunability allows for the manipulation of the qubit's state, which is crucial for quantum computation. The fundamental architecture of a fluxonium qubit consists of three main components:

- A single Josephson junction with Josephson energy E_J.
- A large linear array of additional Josephson junctions, which provides a significant inductive energy E_L. This array is typically made of a large number of junctions, ranging from 10s to 100s.

Fig. 2.1 Charged qubit



- A shunt capacitance C, which provides the charging energy ECEC to the system.

The unique aspect of fluxonium is the balance between these three energies: E_J, E_L, and E_C. The large inductive energy E_L from the array of junctions is critical. It lowers the plasma frequency and makes the energy spectrum of the qubit more anharmonic, which is beneficial for defining and manipulating discrete quantum states. The energy levels of the fluxonium qubit can be tuned externally using magnetic flux. This is achieved by threading a magnetic flux through a superconducting loop formed by the inductive array and the single Josephson junction. The flux tunability allows for the precise control of the qubit's transition frequencies.

The fabrication of a fluxonium qubit involves advanced nanofabrication techniques to create the Josephson junctions and the superconducting circuits. Materials used are typically aluminum or niobium, and the substrate is often sapphire or silicon. Researchers at MIT in 2023 used two fluxonium qubits that exceeded 99.9% accuracy (Zewe 2023). The architecture utilized by the researchers was to have two fluxonium qubits connected via transmon coupler. This is referred to as a fluxonium-transmon-fluxonium (FTF) architecture.

The name transmon is an abbreviation of transmission line shunted plasma oscillation qubit. The transmon is based on the Cooper pair box. The transmon qubit consists of a superconducting island connected to a reservoir via a Josephson junction. A defining feature of the transmon is its large shunt capacitance, which is added to decrease the sensitivity to charge noise. This is achieved by adding a large capacitor in parallel to the Josephson junction(s), which spreads the electron charge over a larger number of Cooper pairs. By diluting the effect of a single electron's charge, the qubit becomes less susceptible to fluctuations in the charge environment, thereby increasing its coherence time. A circuit diagram of a transmon is shown in Fig. 2.2.

The xmon is similar to the transmon. The xmon is basically a tunable transmon, which itself is designed to be less sensitive to charge noise, thus improving coherence times compared to earlier designs like the Cooper pair box. The Xmon qubit features a distinctive "X" shape, which enhances its capacitive coupling and allows for more flexible control line arrangements. This shape facilitates the integration of multiple qubits in a two-dimensional lattice, which is beneficial for scaling up quantum processors. The Xmon is fabricated using superconducting materials such as aluminum on a silicon substrate. The operation of an Xmon qubit involves

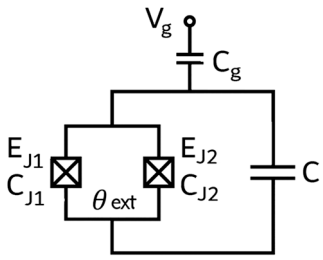


Fig. 2.2 Transmon circuit diagram

manipulating quantum states using microwave pulses. These pulses control the superconducting current that flows through Josephson junctions—key elements in the circuit that exhibit non-linear inductance. The Xmon's design improvements enable efficient tunability and coupling between qubits, crucial for performing quantum logic operations.

Qubits for digital quantum computing and quantum simulation are most commonly fabricated from aluminum wiring (Tc = 1.2 K) and aluminum-amorphous aluminum oxide-aluminum (Al-AlOx-Al) Josephson junctions on either silicon or sapphire substrates. While superconducting qubits can be fabricated using the same design tools and fabrication equipment used to build silicon chips, the premium placed on high coherence necessitates that the specific fabrication steps be modified to eliminate defects that create losses. As a result, the highest-coherence qubits fabricated today, with coherence times of around 100 microseconds, are generally relatively simple devices, using a single layer of metal, rather than the complex processes of 10 metal layers used with the digital silicon or superconducting logic devices in today's classical computers.

Superconducting qubits require milli-Kelvin (mK) temperatures to operate. For digital quantum computing, the qubit operation frequency is typically around 5 GHz, which corresponds to a thermal energy of approximately 250 mK. The qubit must thus be operated at much lower temperatures in order to avoid unwanted thermal excitation of the excited state. This is achieved using commercial 3He/4He dilution refrigerators, which are capable of cooling to sub-10 mK temperatures. On the other hand, for most practical potential uses of a quantum annealer, the qubits will at times operate at frequencies corresponding to thermal temperatures much lower than those achievable with a dilution refrigerator, which make it nearly certain that thermal noise will affect the annealing protocol and drive the system out of its ground state. Quantum annealing will be discussed in Chap. 8.

Modern dilution refrigerators leverage electromechanical pulse-tube coolers to achieve cooling in two stages, one at 50 K and one at 3 K. These are called "dry" refrigerators, as they do not require consumable liquid helium coolant to reach these temperatures. Then, at 3 K, a closed-cycle mixture of helium isotopes, 3He and 4He, is condensed and circulated to achieve cooling through a series of stages at temperatures of 700 mK, 50 mK, and the base temperature of approximately 10 mK. Cooling from room temperature to base temperature can take approximately 36–48 h. But once it has cooled, the refrigerator can maintain those temperatures indefinitely.

Research into improving superconducting qubits proceeds along many different lines. One such approach utilizes ordinary telecommunications technology, optical fiber. Physicists at the National Institute of Standards and Technology (NIST) have measured and controlled a superconducting quantum bit (qubit) using light-conducting fiber instead of metal electrical wires, paving the way to packing a million qubits into a quantum computer rather than just a few thousand. This is an example of how research in diverse areas can have a positive impact on quantum computing.

Flowermon

In 2024, researchers announced a substantial improvement in superconducting quantum computers. Decoherence and noise limit the time for continuous operations of a quantum computer. "The problem is, the advantage of superconducting qubits in enhanced switching time comes at a cost: They are more susceptible to decoherence, which occurs in milliseconds, or even faster. To mitigate this issue, scientists typically resort to meticulous adjustments of circuit configurations and qubit placements—with few net gains."[1]

In a study published in the *Physical Review Letters*, researchers discovered that if the angle between the crystal lattices of two superconducting cuprate sheets is 45 degrees, the qubit exhibits more resilience to external disturbances compared to conventional designs based on materials like niobium and tantalum. Cuprate sheets are copper oxide ceramics that display superconducting properties, but at higher temperatures than other superconductors. Cuprates are classified as Type II superconductors, which means they can sustain a mixed state where magnetic fields partially penetrate the material in the form of vortices. This property allows them to carry large currents and generate strong magnetic fields, which are useful for applications like magnetic resonance imaging (MRI) and maglev trains.

Van-der-Waals assembly enables the fabrication of novel Josephson junctions featuring an atomically sharp interface between two exfoliated and relatively twisted $Bi_2Sr_2CaCu_2O_{8+x}$ flakes. In a range of twist angles around 45°, the junction provides a region where the interlayer two-Cooper pair tunneling dominates the current-phase relation. This creates the qubit called the *flowermon*.

Van der Waals assembly refers to a method of constructing layered materials and structures through the stacking of two-dimensional (2D) materials, driven by van der Waals forces. These forces, though relatively weak compared to chemical bonds, are significant in the context of 2D materials like graphene, transition metal dichalcogenides (TMDCs), hexagonal boron nitride (hBN), and others. The process leverages the attraction between these layers without the need for covalent or ionic bonding. By stacking different 2D materials, it is possible to create heterostructures with unique properties not found in the individual layers. These properties include varying electronic, optical, and mechanical characteristics, which can be tailored by choosing specific combinations of layers, their sequence, orientation, and rotational alignment.

Specific Implementations

A range of companies have produced superconducting qubit quantum computers. It can be educational to review some of these implementations. In the following subsections a few of the producers of superconducting quantum computers will be explored, including their specific quantum processors.

[1] https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.132.017003

IBM

IBM has been a prominent figure in superconducting quantum computers. They have produced several different superconducting quantum processors. Improving coherence times has been a major focus of IBM. Another focus has been achieving larger numbers of qubits.

Q-Series

IBM had several processors in the Q series. Q-17 was a 17-qubit processor announced in May 2017. IBM Q53 was a 53-qubit processor announced in October 2019. The IBM Q53 has a unique qubit layout designed to optimize performance and error rates. The arrangement allows for more efficient quantum operations and enhances the potential for running complex quantum algorithms.

Eagle

The IBM Eagle boasted 127 qubits and marked a change in IBM's naming conventions. From this processor forward, IBM favored using bird names. The architecture of the Eagle processor is designed to minimize errors and enhance computational capabilities. It employs heavy hexagonal qubit connectivity, which helps in error correction and improves the scalability of the system. This design is part of IBM's approach to achieving a fault-tolerant quantum computer.

Osprey

At the Quantum Computing Summit on November 9th IBM announced the 433 Qubit OSPREY. The control electronics run in a cryo-CMOS controller chip that operates at approximately 4K. Rather than the 100 watts per qubit they previously needed, the new control chip uses only 10 milliwatts per qubit. IBM uses a quantum computing speed metric known as circuit layer operations per second (CLOPS), the company has gone from 1400 to 15,000 CLOPS with its best systems. Right now, the median coherence time for Osprey is around 70–80 microseconds. Uses transmon superconducting qubit architecture. Recall that transmons are superconducting charge qubits designed to be less sensitive to noise. More on transmons on the next slide. Qubits are coupled to superconducting bus resonators and read out through multiplexed flex RF cables.

Condor

Announced to the public in December 2023, the Condor is a 1121 superconducting qubit quantum processor. It boasts a 50% increase in qubit density and over a mile of high-density cryogenic flex IO wiring within a single dilution refrigerator. Along with increasing the number of qubits, IBM is focusing on improving qubit fidelity, which involves reducing error rates in quantum operations. High-fidelity qubits are essential for performing reliable and practical quantum computations, particularly as the systems scale up.

IBM researchers state "One of the big focuses of Condor was simplifying the wiring outside the qubit chip—untangling the I/O as much as possible on the chip to simplify everything else inside of the dilution refrigerator, To accomplish that, the multi-level wiring we used on Eagle and Osprey was expanded from three levels (ground-signal-ground or GSG) to five levels (GSGSG), allowing routes to cross."[2]

Heron

IBM also has the Heron processor which will be used in stacking multiple 133-qubit units together. Heron, unlike Eagle, Osprey, and Condor, is built on a new chip architecture. This architecture includes tunable couplers. The concept behind Heron is to have what IBM has termed "utility scale quantum processors." This essentially means these are designed for real world use.

IBM has announced a roadmap of their plans for the next several years. It should be noted that their previous strategic milestones have been achieved, on the dates planned. This bodes well for IBM's future plans. That roadmap is shown in Fig. 2.3.

While IBM is only one company, and it is not the purpose of this book to endorse a specific company, their accomplishments are noteworthy in the realm of superconducting quantum computing.

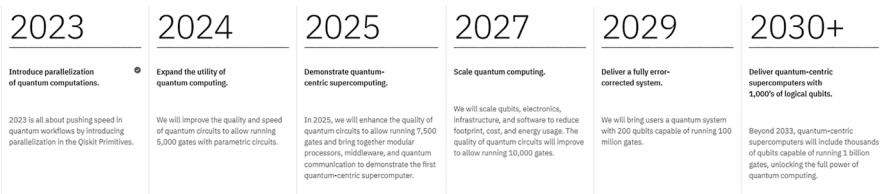


Fig. 2.3 IBM roadmap

[2] https://www.allaboutcircuits.com/news/closer-look-at-ibms-heron-and-condor-quantum-processors/

Google

Along with IBM, Google is one of the most prominent creators of superconducting quantum processors. Examining a few of their processors can be quite useful.

Bristlecone

Google Bristlecone was a quantum processor announced by Google in March 2018. It was designed as part of Google's ongoing efforts to develop scalable and fault-tolerant quantum computing systems. Bristlecone was designed to have a large number of qubits arranged in a two-dimensional lattice structure. The qubits were implemented using superconducting circuits, which offer relatively long coherence times and the ability to fabricate large arrays of qubits. One of the main goals of the Bristlecone project was to demonstrate scalability in quantum computing systems. Google aimed to show that it was possible to increase the number of qubits while maintaining low error rates, paving the way for building larger, more powerful quantum processors in the future. Bristlecone used transmons for the superconductor.

Sycamore

Sycamore gained significant attention in October 2019 when Google claimed to have achieved a milestone known as "quantum supremacy" using the Sycamore processor. The task performed by the Sycamore processor to demonstrate quantum supremacy was sampling from the output distribution of a randomly generated quantum circuit, specifically designed to be computationally hard for classical computers to simulate. This task, known as random quantum circuit sampling, is a benchmark for assessing the capabilities of quantum computers.

The Sycamore processor comprises an array of interconnected qubits arranged in a two-dimensional lattice structure. Much like Bristlecone, Sycamore uses transmon superconducting/Each qubit can interact with its neighboring qubits through programmable couplers, allowing for the execution of quantum algorithms. In the qubit lattice, each qubit is typically coupled to its nearest neighbors. This connectivity pattern allows for the implementation of quantum gates and interactions between neighboring qubits, which are essential for performing quantum computations. In addition to qubits, the Sycamore processor includes control and readout lines that are used to manipulate and measure the quantum states of the qubits. Control lines provide microwave pulses that control the qubit operations, such as single-qubit gates and two-qubit gates. Readout lines are used to detect and read out the quantum states of the qubits at the end of a computation. The Sycamore processor also features programmable couplers that allow for tunable interactions between pairs of qubits. These couplers can be adjusted to control the strength and duration of interactions between qubits, enabling the execution of quantum algorithms.

Rigetti

Rigetti has been a prominent participant in the development of quantum computing for many years.[3] They now offer a 9-qubit quantum processing unit called the Novera, for public purchase.[4] The system is meant to operate with dilution refrigerators. Rigetti offers access to its quantum processors via the Quantum Cloud Services platform. This platform integrates classical computing resources with quantum processors, enabling hybrid quantum-classical algorithms and applications. Rigetti provides the Forest SDK, which includes tools and libraries for developing quantum algorithms. This SDK supports a range of programming languages and provides simulators to test quantum programs.

The Acorn was first released in December of 2018 and is based on transmons. This processor has 19 qubits and a 98.63% fidelity. The Agave processor was released in June 2018 and has 9 qubits with a 96% fidelity. Rigetti also has the Aspen series beginning with Aspen 1 released in November 2018 with 16 qubits continuing to the Aspen-M-3 released in December 2022 with 80 qubits and achieves 99.9% fidelity with single qubit gates and 94.7% to 95.1% fidelity with 2 qubit gates.

Riken

Riken is a Japanese research institute. They conduct research in a wide range of areas including quantum computing.[5] In March of 2023, in cooperation with Fujitsu, announced a 64-qubit superconducting quantum computer.[6] Their website contains a great deal of information about their ongoing quantum computing research. The following excerpt is an example "We are also conducting research on superconducting quantum circuits while using a cluster state. In this research, we succeeded for the first time in creating a one-dimensional cluster state in the time domain along a superconducting path [arXiv:2105.08609]. By reusing the qubits repeatedly in the time domain, we successfully created a 4-qubit equivalent linear cluster state from only two physical qubits with fidelity as high as 59%. We also examined the expectation value for projector witness and confirmed that the created four qubits are in a genuine multipartite entanglement (GME) state. This shows that an entangled state of qubits that exceeds the number of physical superconducting qubits can be created. The complexity of the qubit's spatial domain is thus reduced, making it possible to create, in the future, a three-dimensional cluster that allows for error

[3] https://www.rigetti.com/

[4] https://www.rigetti.com/novera

[5] https://www.riken.jp/en/about/

[6] https://www.fujitsu.com/global/about/resources/news/press-releases/2024/0315-01.html

correction. We have prototyped a quantum chip that creates an 18-bit cluster state using a quasi-two-dimensional network."[7]

China's Zuchongzhi Processor

China has traditionally been focusing on photonic quantum computing (see Chap. 3). However, they also have developed superconducting qubit quantum computers. Chinese 66-qubit Zuchongzhi 2 processor has outperformed Googles Sycamore processor on several algorithms. The Zuchongzhi 2 is photon based and is much faster than the superconducting qubit-based processors.[8] Note: Zu Chonzhi was an astronomer and mathematician circa 500 AD. This year, China announced a 176-qubit version of Zuchongzhi. This project was spearheaded by University of Science and Technology of China (USTC).[9] The Zuchongzhi processor uses its own gate, an iSWAP like gate:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i(\Delta_+ + \Delta_-)\cos\theta} & -ie^{i(\Delta_+ - \Delta_{-,\text{off}})\sin\theta} & 0 \\ 0 & -ie^{i(\Delta_+ + \Delta_{-,\text{off}})\sin\theta} & e^{i(\Delta_+ - \Delta_-)\cos\theta} & 0 \\ 0 & 0 & 0 & e^{i(2\Delta_+ - \phi)} \end{bmatrix}$$

Now that China has ventured into superconducting quantum computing, it is reasonable to expect to see more advances from them in this arena.

Xiaohong

The Chinese Academy of Sciences has announced the Xiaohong quantum processor with 504 qubits.[10] The word Xiaohong means *morning rainbow*. Gong Ming, a researcher at the center, said that the primary objective of the Xiaohong chip is not for enhancing computing power or achieving quantum supremacy. The main purpose of the chip is to foster the development of large-scale quantum computing measurement and control systems. The first Xiaohong chip was delivered to QuantumCTek, a company that manufactures quantum secure communication,

[7] https://rqc.riken.jp/en/

[8] https://quantumzeitgeist.com/chinas-176-qubit-quantum-computing-platform-zuchongzhi-goes-global/

[9] https://quantumcomputingreport.com/chinese-upgrade-zuchongzhi-2-quantum-computer-from-66-to-176-qubits/

[10] https://www.chinadaily.com.cn/a/202404/26/WS662b15dfa31082fc043c431e.html

quantum computing and quantum measuring. The company will use the chip to test their kilo-qubit measurement and control system. There are plans to also offer access to this chip via cloud resources.

Research

There is an extensive body of research publications regarding superconducting qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

In 2023, MIT published research on a new approach to quantum operations with superconducting qubits.[11] MIT researchers have introduced a new superconducting qubit design using fluxonium qubits. This new architecture notably improves the accuracy of quantum operations, achieving gate operations with fidelity exceeding 99.9% for two-qubit gates and 99.99% for single-qubit gates. The new circuit utilizes fluxonium qubits, which are known for their longer lifespans compared to the more commonly used transmon qubits. This longevity allows for higher fidelity in operations and greater robustness against errors. Both fluxonium and transmon qubits were discussed earlier in this chapter.

The architecture incorporates a special coupling element between two fluxonium qubits, enhancing their ability to perform logical operations with high accuracy. This setup helps suppress unwanted background interactions that can introduce errors. The high fidelity of operations achieved with this new architecture surpasses the threshold required for implementing effective quantum error-correcting codes, reducing the overhead needed for such corrections.

An article published in Nature communications in 2022 focused on reducing quasiparticles and charge noise.[12] The research focuses on reducing quasiparticles, which are broken Cooper pairs that can lead to decoherence in qubits. Techniques such as downsizing the qubit and adding a metallic cover are explored to control quasiparticle generation. A flip-chip design is used to shape the electromagnetic environment of the qubit, which helps inhibit quasiparticle poisoning by blocking stray photon absorption. A significant achievement discussed is the low charge-parity switching rate, less than 1 Hz, achieved in aluminum qubits. This indicates improved stability against discrete charging events that can cause operational errors. Detailed methods for device fabrication and measurement setup are provided in the article, highlighting the intricate processes involved in creating and testing these advanced superconducting qubits.

[11] https://news.mit.edu/2023/new-qubit-circuit-enables-quantum-operations-higher-accuracy-0925

[12] https://www.nature.com/articles/s41467-022-34727-2

Conclusions

Superconducting quantum computing is a promising area of quantum computing research. Many diverse groups are working on various implementations of superconducting quantum computing. These groups include teams in the United States, Japan, Europe, and China. Superconducting qubits are very scalable and designable, as well as being easy to control. However, superconducting qubits do experience decoherence faster than some other physical qubit implementations. Superconducting qubits are also more error prone.

Chapter 3
Photonic Quantum Computing

Introduction

Photonic quantum computing is a branch of quantum computing that utilizes photons (particles of light) as the basic unit of information, called qubits, instead of traditional electronic bits. In this paradigm, information is encoded in the quantum properties of photons, such as their polarization, phase, or path. Photons can be manipulated to represent qubits. For example, the polarization state of a photon can be used as a qubit. Horizontal polarization can represent 0, while vertical polarization can represent 1. By manipulating the polarization of photons, quantum operations can be performed.

Photon Details

Photons are fundamental particles of light and other forms of electromagnetic radiation. They are the carriers of electromagnetic force and have zero rest mass, meaning they always travel at the speed of light in vacuum, denoted by "c" (~299,792 km/s). Photons are essentially discrete bundles of electromagnetic energy. They exhibit both wave-like and particle-like properties, depending on how they are observed or measured. When treated as particles, photons are described as quanta of electromagnetic radiation, carrying energy proportional to their frequency according to Planck's relation $E = hf$, where E is energy, h is Planck's constant, and f is frequency. Planck's constant is $6.62607015 \times 10^{-34}$ joule-hertz−1 (or Joule-seconds). Rather than being expressed in joules, you will frequently see Plank's constant described as $6.62607015 \times 10^{-34}$ m^2 kg/s. Photons can interact with matter through various processes, including absorption, emission, and scattering. These interactions play crucial roles in phenomena such as the photoelectric effect, where photons striking a

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_3

material surface can eject electrons, and in the formation of images in cameras and the human eye.

Photons are often described as excitations of the electromagnetic field in quantum field theory (QFT). In QFT, the photon field is quantized, and particles are viewed as excitations of these fields. In Quantum Field Theory, photons are described using creation and annihilation operators. The creation operator a† creates a photon, while the annihilation operator a annihilates a photon. These operators satisfy commutation relations:

$$\left[a, a\dagger\right] = 1$$

where [·,·] denotes the commutator.

Photons can have different polarization states, which are described using polarization vectors. For example, for linear polarization along the x-axis, the polarization vector can be written as ex. These mathematical concepts form the basis for understanding and describing the behavior of photons in quantum mechanics and quantum field theory.

As was mentioned previously in this chapter, photons can be emitted or absorbed. The emission can be spontaneous or stimulated. In spontaneous emission, an atom or molecule transitions from a higher energy state to a lower energy state, emitting a photon in the process. This occurs without any external stimulus and is governed by the laws of quantum mechanics. The probability of spontaneous emission depends on the transition probability between the initial and final states, as described by the Einstein coefficients. In stimulated emission, an incoming photon interacts with an excited atom or molecule, causing it to transition to a lower energy state and emitting a second photon with the same frequency, phase, polarization, and direction as the incoming photon. This process is the basis of laser operation, where the emitted photons stimulate further emission, leading to coherent light amplification.

When an electron in an atom or molecule transitions from a higher energy level to a lower one, it can emit a photon. The energy of the photon corresponds to the energy difference between the two levels. Absorption can occur when a photon encounters an atom. It may be absorbed if its energy matches the energy difference between the atom's energy levels. Absorption occurs when the photon transfers its energy to the system, promoting it from a lower energy state to a higher energy state. The probability of absorption depends on factors such as the energy of the photon and the transition probabilities between the initial and final states. Another type of absorption is resonance absorption. Resonant absorption occurs when the frequency of the incoming photon matches the natural frequency of a transition between two quantum states in the absorbing medium. This leads to enhanced absorption and is exploited in various applications, including spectroscopy and optical filters. Absorption and emission are shown in Fig. 3.1.

There are other scenarios which can also lead to the emission of photons. When particle-antiparticle pairs annihilate, such as an electron and a positron, the result is the creation of photons. For example, electron-positron annihilation typically results in the creation of two photons, each carrying away the energy equivalent to the mass

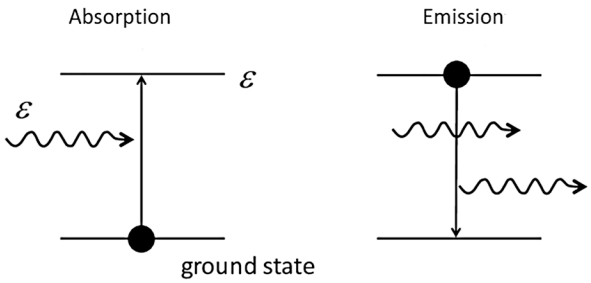


Fig. 3.1 Absorption and emission

and kinetic energy of the annihilating particles. Accelerating or decelerating electric charges emit photons. This is the principle behind antennas for radio waves and synchrotron radiation, where electrons moving at relativistic speeds in magnetic fields emit high-energy photons.

Similarly, photons can also be annihilated. In the presence of a strong electromagnetic field, such as near a heavy nucleus, a high-energy photon can be annihilated to create a particle-antiparticle pair (e.g., an electron and a positron). This requires the photon to have enough energy to match at least the combined rest mass energy of the two particles it creates, which is at least 1.022 MeV (the rest mass energy of an electron plus that of a positron). Photons can also be effectively annihilated through scattering processes, such as Compton scattering, where a photon collides with an electron and transfers part of its energy to the electron, emerging with reduced energy and changed direction.

For those readers not familiar with Compton scattering, more detail is provided here. Compton scattering is a quantum mechanical process where a photon (a particle of light) collides with a charged particle, usually an electron, causing the photon to change direction and reduce its energy. This process was discovered by Arthur H. Compton in 1923, and it provided important evidence supporting the particle theory of light and the concept of the photon. Compton's discovery was crucial for the development of quantum mechanics, and he received the Nobel Prize in Physics in 1927 for his work. Compton scattering is significant because it demonstrates that light cannot be explained purely as a wave phenomenon; it also exhibits particle-like characteristics. This process occurs predominantly at X-ray and gamma-ray wavelengths but can happen with any photon energy depending on the context. In Compton scattering, when a photon strikes a loosely bound or free electron, it transfers part of its energy and momentum to the electron, causing the photon to scatter with reduced energy and therefore a longer wavelength. The change in the photon's wavelength depends only on the angle of scattering and is independent of the material's properties. This phenomenon can be described through conservation laws:

Conservation of Energy: The total energy (the sum of the photon's energy and the electron's kinetic energy) must be conserved.
Conservation of Momentum: The total momentum must also be conserved.

The Compton equation describes the change in the wavelength of a photon and is shown here:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_c c}\left(1 - \cos\theta\right)$$

In the preceding equation:

λ = the initial wavelength of the photon
λ' = the wavelength of the photon after scattering
h = the Planck constant
m_c = the mass of the electron
c = the speed of light
θ = the angle at which the photon is scattered relative to its original direction.
$\frac{h}{m_c c}$ is known as the Compton wavelength of an electron. Approximately 2.43×10^{-12} m.

From a quantum mechanical perspective, Compton scattering can be seen as the collision between two particles: a photon and an electron. The initial and final states of the system can be described using quantum states, with the scattering process modeled using Feynman diagrams that depict the exchange of momentum and energy.

Details on Photonic Qubits

As with all physical implementations of qubits, photonic qubits require quantum gates. Quantum gates are the building blocks of quantum algorithms. In photonic quantum computing, these gates manipulate the state of qubits, enabling operations such as superposition, entanglement, and interference. Examples of photonic quantum gates include the Hadamard gate for creating superpositions and the CNOT gate for entangling qubits. Similar to other quantum computing architectures, measurement in photonic quantum computing collapses the superposition of qubits into classical states. This allows the extraction of classical information from quantum systems.

Photons can be entangled, a phenomenon where the state of one photon is correlated with the state of another, even if they are physically separated. This property is crucial for performing certain quantum algorithms, such as quantum teleportation and quantum key distribution. Photonic quantum computing faces several challenges, including the difficulty of generating and manipulating single photons, as well as efficiently detecting and measuring them. Additionally, photon loss and decoherence (the loss of quantum information due to interactions with the environment) are significant obstacles that researchers are working to overcome.

Photons are commonly used for creating physical qubits, with the data (1 or 0) determined by the polarization. The horizontal polarization is |0> and vertical is |1>. The polarization method is a common method for using photons for qubits. This is the essence of using polarization for encoding qubits.

Photons can be described as having a horizontal or vertical polarization. Some sources refer to a right or left circular polarization. In general, polarization specifies a geometrical orientation for a transverse wave. A transverse wave is a wave with oscillations that are perpendicular to the direction of the wave. The term perpendicular, in this instance, is synonymous with transverse. The motion of a transverse wave can be described in the following formula:

$$s(p,t) = Au \sin\left(\frac{t - (\rho - 0)\frac{\mathrm{d}}{v}}{T} + \varphi \right)$$

Where in that formula, the values are:

d is the direction of propagation
o is a reference point in the medium of propagation
A is the wave amplitude.
v is the velocity of propagation
T is the period
φ is the phase at the reference point o
p is a location/point
t is the time

This is primarily a description of linear polarization. Linear polarization refers to the orientation of oscillations of electromagnetic waves, such as light, along a particular direction. In linearly polarized light, the electric field vector oscillates in a single plane as the wave propagates through space. This orientation is typically described in terms of the angle between the direction of polarization and a reference axis.

Circular polarization refers to the polarization state of electromagnetic waves, such as light, where the electric field vector rotates in a circular pattern as the wave propagates through space. Unlike linear polarization, where the electric field oscillates along a straight line, in circular polarization, the electric field vector traces out a helical pattern. Circular polarization allows for multiple independent displacement directions along with the direction d. While the light is commonly used in discussions of polarization, any electromagnetic wave can have polarization. There are also forms of polarization other than circular or linear. For example, elliptical polarization. Any electromagnetic wave that is polarized such that the tip of the field vector defines an ellipse that is in a fixed intersecting plan, and perpendicular to the direction of propagation is elliptically polarized.

Circularly polarized light is prevalent in nature, with examples including the polarization of light reflected from certain surfaces, the polarization patterns of light emitted by certain animals (e.g., some insects and cephalopods), and the polarization of light emitted by astronomical sources. Circular polarization has diverse applications in fields such as optical communications, remote sensing, and 3D cinema technology. It is particularly useful in scenarios where the orientation of linear polarization may vary unpredictably, such as in optical communication systems affected by atmospheric turbulence.

Circularly polarized light can rotate in either a clockwise (right-handed) or counterclockwise (left-handed) direction as viewed from the source. The direction of rotation is determined by the orientation of the electric field vector as it spirals along the direction of propagation. The electric field vector of circularly polarized light completes one full rotation as the wave travels one wavelength in the direction of propagation. This helical motion can be visualized as a corkscrew-like pattern. Circular polarization can arise through various mechanisms, including the differential absorption or phase retardation of light waves with different circular polarizations. This phenomenon is exploited in techniques such as circular dichroism spectroscopy for studying molecular chirality and structure. Circularly polarized light can be generated using specialized optical components, such as wave plates or quarter-wave plates, which introduce a phase delay between the orthogonal components of linearly polarized light. Additionally, certain materials and optical phenomena naturally produce circular polarization.

Another method used with photons is to encode the qubit using time-bin encoding. The process involves having a single photon be processed through a Mach-Zehnder interferometer. This interferometer is a device that takes two beams that are derived from splitting a single light source and determines the relative phase shift variations between them. When a photon enters the interferometer, it will be guided along one of two paths. One path is longer than the other, and the difference between the two paths must be longer than the photon's coherence length. Coherence length is the propagation distance over which a coherent electromagnetic wave can maintain a particular degree of coherence. The photon takes one of the two paths. The shorter path (i.e., the earlier arrival of the photon) designates the state |0> and the longer path designates state |1>. A schematic of a Mach-Zehnder interferometer is shown in Fig. 3.2.

The incoming light beam is split into two beams by the beam splitter. Traditionally, this splitter is a partially silvered mirror that reflects some of the incident light and transmits the rest. After passing through the beam splitter, the light splits into two paths. Let us call these paths the "upper" and "lower" paths for clarity. After passing through the beam splitter, the light splits into two paths. Let us call these paths the "upper" and "lower" paths for clarity. The two paths converge again at the second beam splitter or recombiner. This component recombines the two light paths. When the two paths recombine, they can interfere constructively or destructively depending on the relative phase of the light in each path. If the phase difference between the two paths is an integer multiple of the wavelength of the light, constructive interference occurs, resulting in a bright spot at the output. If the phase difference is

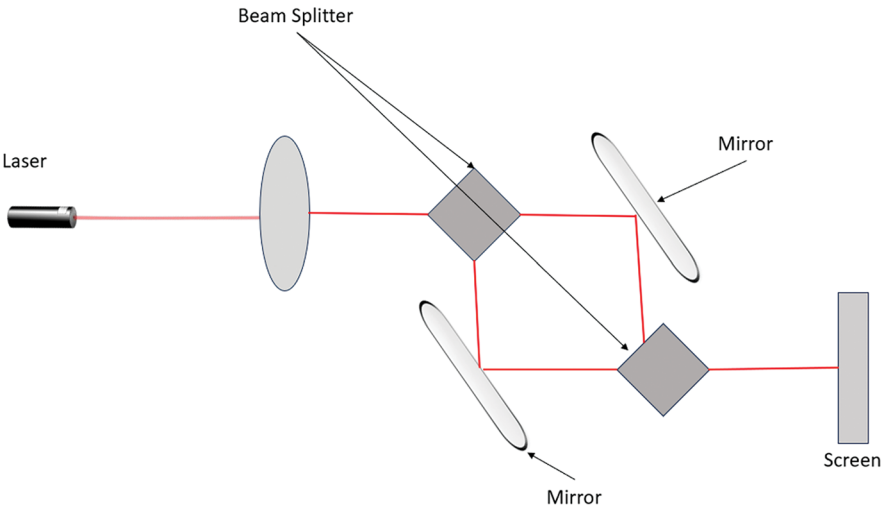


Fig. 3.2 Mach Zehnder interferometer

a half-integer multiple, destructive interference occurs, resulting in a dark spot at the output. By introducing a phase shift in one of the paths (for example, by inserting a transparent material or altering the length of one path), the interference pattern at the output changes. This change in interference pattern allows for the measurement of the phase shift induced by the sample or external conditions.

One can model a photon going through the interferometer by assigning a probability amplitude to each of the two possible paths: the "lower" path which starts from the left, goes straight through both beam splitters, and ends at the top, and the "upper" path which starts from the bottom, goes straight through both beam splitters, and ends at the right. The quantum state describing the photon is therefore a vector.

A variation of using photons is Linear Optical Quantum Computing (LOQC). In LOQC, qubits are typically represented by the states of single photons or entangled pairs of photons. The polarization, path, or mode of the photons can be used as qubit encodings. Quantum interference plays a crucial role in LOQC. By manipulating the paths of photons using beam splitters and phase shifters, interference effects can be exploited to perform quantum computations. Interference allows for the constructive and destructive interference of probability amplitudes, enabling complex quantum operations to be implemented.[1, 2] LOQC faces challenges such as photon loss, noise, and the lack of efficient non-linear optical elements for implementing certain types of quantum gates. However, LOQC also offers advantages, including scalability and compatibility with existing optical communication technologies.

[1] https://arxiv.org/abs/quant-ph/0512071

[2] https://inspirehep.net/literature/773113

Additionally, photon-based qubits are inherently robust against decoherence due to their low interaction with the environment.

In 2020, researchers at Harvard University proposed a novel method for using photons for qubits. The qubit would be a single atom in their proposed approach, with the atom's photons as information carriers. In order to use photons in quantum computation, the photons must interact; however, under normal conditions, photons do not interact with themselves. The Harvard team proposed using a mirror to reflect photons produced by the atom back to the atom so they would interact with the atom. As of this writing, this is a theoretical proposal and has not yet been realized in a physical device.

Specific Implementations of Photonic Computing

In addition to understanding the generalities of photonic computing, it is informative to understand specific implementations of quantum computing. Various companies have implemented quantum computing using photons, and a better understanding of their implementations would help in elucidating the concepts of photonic quantum computing.

China's Jiuzhang Processor

The Jiuzhang processor, also known as the "Light-Matter Interactions Enhanced by Quantum Decoherence" processor, is a photonic quantum computing device developed by a team of researchers in China. It gained significant attention in 2020 due to claims of achieving quantum supremacy, which refers to the ability of a quantum computer to solve certain problems faster than classical computers. The Jiuzhang processor is named after an ancient Chinese mathematical text Jiuzhang Suanshu, also known as the "Nine Chapters on the Mathematical Art." Jiuzhang Suanshu, is one of the most influential ancient Chinese mathematical texts. It was compiled over several centuries, with its earliest known version dating back to the 1st century BCE during the Han Dynasty. The text consists of nine chapters, each addressing various mathematical topics and practical problems. The text is notable for its emphasis on practical applications and problem-solving techniques, reflecting the mathematical knowledge and skills needed for everyday life and various professions during ancient times in China. It served as a foundational text for the development of mathematics in East Asia and had a significant influence on later mathematical traditions in China and neighboring regions.

The core of the Jiuzhang processor is an optical interferometer, a device that combines multiple light paths to generate interference patterns. The interferometer in Jiuzhang is highly complex and involves multiple modes of light interacting with each other. The interferometer used in the Jiuzhang processor is a crucial

component that enables the manipulation and interaction of photons to perform quantum computations, specifically for the boson sampling task. Interferometers are devices that combine multiple light paths and generate interference patterns, exploiting the wave nature of light to achieve various optical effects. This interferometer features a highly complex optical setup consisting of multiple beam splitters, phase shifters, and detectors. These optical elements are precisely arranged to control the paths and phases of photons as they traverse the interferometer. In the Jiuzhang interferometer, beam splitters are used to split incoming photons into different paths, creating interference between the photons that recombine at later stages. Phase shifters are components that introduce controlled phase differences between the paths of the split photons. By adjusting the phase of individual photons, researchers can manipulate the interference patterns generated by the interferometer.

Inside the interferometer, photons interact with each other through the process of interference. Depending on their relative phases and paths, photons can either reinforce each other (constructive interference) or cancel each other out (destructive interference). After traversing the interferometer, the photons are directed to output ports where they are detected by photon-counting devices. The detection pattern, representing the spatial distribution of photons at the output ports, provides information about the quantum state of the system. Figure 3.3 is a schematic of the Jiuzhang processor:

The Jiuzhang processor performs a specialized quantum computing task known as boson sampling. Boson sampling involves sending identical photons through a network of beam splitters and measuring the output distribution. This task is believed to be computationally hard for classical computers to simulate. Boson sampling is a quantum computational task that aims to demonstrate the computational advantage of quantum systems over classical computers. It involves sending indistinguishable photons through a network of optical elements, such as beam splitters, and measuring the output distribution of photons at the output ports. The output distribution represents the probabilities of different photon configurations and is difficult to simulate efficiently using classical computational methods. In boson sampling, identical photons are used as input to the optical setup. These photons are indistinguishable, meaning that there is no way to tell them apart based on their properties

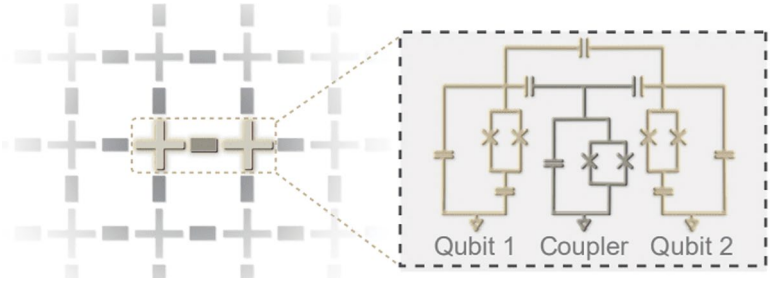


Fig. 3.3 Jiuzhang processor

such as wavelength, polarization, or arrival time. The heart of the boson sampling experiment is an optical interferometer, which consists of a network of beam splitters and phase shifters arranged in a specific configuration. The interferometer is designed to split, recombine, and interfere with photons as they propagate through the optical paths. Boson sampling is proposed as a task that can demonstrate the quantum advantage of quantum systems over classical computers. While simulating boson sampling using classical algorithms is believed to be difficult, implementing it with quantum systems, such as photonic quantum computers, could potentially solve the problem efficiently. Overall, boson sampling represents a significant quantum computational task that serves as a benchmark for assessing the capabilities of quantum systems and exploring the potential for achieving quantum computational advantages over classical methods.

In December 2020, researchers at the University of Science and Technology of China reported that the Jiuzhang processor had achieved quantum supremacy. They claimed that Jiuzhang performed a boson sampling task in about 200 s, which they argued would take the world's fastest supercomputers tens of thousands of years to complete. The claim of quantum supremacy by Jiuzhang has been met with some skepticism and criticism from the scientific community. Some experts argue that boson sampling is not a universal demonstration of quantum supremacy and that its practical relevance remains uncertain. Additionally, there have been debates about the difficulty of simulating the Jiuzhang experiment on classical computers.

The latest version of Jiuzhang, as of 2024, is the Jiuzhang 3.0 which has 255 detected photons. The original Jiuzhang had 76 and Jiuzhang 2.0 had 113. A paper published in 2023 claimed that with 372 qubits, the Jiuzhang could break 2048-bit RSA keys.[3] Chinese state media claims that Jiuzhang 3.0 is a million times faster than Jiuzhang 2.0 at solving Gaussian boson sampling problem. Experimental data from Jiuzhang 3.0 is available online.[4]

Xanadu Quantum Technologies

Xanadu is a Canadian company that, among other things, produces photonic quantum computers.[5] Their quantum hardware solution is dubbed the X-Series. Xanadu's particular approach to photonic quantum computing has been the subject of an article in the journal Nature.[6] Xanadu's X-Series utilizes Gaussian Boson Sampling (GBS). Their website describes this as follows "Gaussian Boson

[3] https://www.defenseone.com/technology/2023/01/china-about-destroy-encryption-we-know-it-maybe/382041/

[4] https://quantum.ustc.edu.cn/web/en/node/1121

[5] https://www.xanadu.ai/

[6] https://www.nature.com/articles/s41586-021-03202-1

Sampling (GBS) is a special-purpose model of photonic quantum computation, first introduced in Ref. [1]. In its most general form, GBS consists of preparing a multi-mode Gaussian state and measuring it in the Fock basis. It differs from universal photonic circuits only because it does not employ non-Gaussian gates and it restricts measurements to the Fock basis. In general, the output distribution of a GBS device cannot be simulated in polynomial time with classical computers [1] [2]. Applications of GBS aim to harness the computational power unique to GBS to perform useful tasks."[7]

Gaussian Boson Sampling (GBS) is a quantum computational task similar to traditional Boson Sampling, but with some differences in the quantum states and measurements involved. While traditional Boson Sampling involves sampling from the output distribution of a linear optical network with non-Gaussian input states, GBS uses Gaussian states as input and relies on linear optics and photodetectors for measurement. In GBS, the input states are Gaussian states, which are quantum states characterized by Gaussian probability distributions in phase space. These states are relatively easy to prepare and manipulate using linear optical elements and are often generated using techniques such as squeezed states or coherent states. Similar to traditional Boson Sampling, GBS relies on a linear optical network composed of beam splitters and phase shifters. The input Gaussian states are fed into the optical network, where they undergo transformations determined by the network's parameters (e.g., beam splitter transmission coefficients and phase shifts).

After passing through the optical network, the output states are measured using photodetectors. Unlike traditional Boson Sampling, which typically involves single-photon detectors, GBS measurements can be performed with photodetectors that can resolve photon number distributions, such as photon-number-resolving detectors or bucket detectors. The output distribution of GBS represents the probabilities of different photon number configurations at the output ports of the optical network. This distribution is determined by the properties of the input Gaussian states and the parameters of the optical network. Sampling from this output distribution is computationally hard for classical computers, making GBS a potential candidate for demonstrating quantum advantage.

The process used by Xanadu utilizes a small (10 mm × 4 mm) photonic chip that generates squeezed light in up to 8 separate optical modes. There software interface for simulations is called *Strawberry Fields* and integrates quite well with Python.[8] For programming actual quantum computers, the product is *Penny Lane.* For those readers not understanding the references, both names refer to lines in songs by the Beatles.

[7] https://strawberryfields.ai/photonics/concepts/gbs.html

[8] https://strawberryfields.ai/photonics/demos/squeezer_tests.html

United Kingdom's ORCA

ORCA is a company with headquarters in London, but offices in Kakow, Poland; Austin, Texas; Seattle, Washington; and Toronto, Canada.[9] One of the issues that ORCA focuses on is reducing photon loss. Citing the ORCA website "In photonic quantum computers, measurement modules serve as the counterparts to resource state generators, functioning as an 'engine' that consumes resource states to progress a computation. However, these measurement modules require specific types of resource states, just as different engines need different types of fuel. ORCA achieved a breakthrough by demonstrating that some of the burden can be shifted to how we measure resource states. By adding a modest level of complexity into the measurement protocol, traditionally regarded as the 'easy' component of the computer, we can unlock a category of simpler resource states. This advancement does not compromise performance but rather redefines it, like designing a new model of an engine capable of operating on a cheaper type of fuel. Overall, with our state-of-the-art linear-optical measurement protocol, ORCA can unlock a new class of resource states and improve the overall photon-loss tolerance of a photonic quantum computer."[10]

PsiQuantum

It may be useful to simply cite what PsiQuantum says about themselves "PsiQuantum's mission is to build and deploy the world's first useful quantum computer. The company was founded on the premise that commercially valuable quantum computing will require error correction, and therefore very large-scale systems. We believe that we have a fast and feasible path to large-scale fault-tolerant systems, based largely on leverage of existing technology – including high-volume semiconductor manufacturing, packaging, and high-power cryogenic systems."[11]

As one example of their research, they are working on methods to better break classical asymmetric cryptography with less quantum computing resources. One of their published papers is entitled "How to computer a 256-bit elliptic curve private key with only 50 million Toffoli gates."[12] The have also been working on how to use quantum computers to model chemical simulations, particularly for lithium-ion batteries.[13]

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[9] https://orcacomputing.com

[10] https://orcacomputing.com/orca-research-illuminates-path-to-fault-tolerance-with-photon-loss-resistant-protocols-using-simpler-resource-states/

[11] https://www.psiquantum.com/about

[12] https://arxiv.org/pdf/2306.08585

[13] https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.4.023019

Quantum Computing Inc.

Quantum Computing Inc., Dirac-3 uses nanophotonic technology to solve complex optimization problems by encoding these problems into its photonic architecture. This system is designed to operate at room temperature and requires no special infrastructure, making it accessible and affordable. Unlike traditional quantum computers that use qubits, Dirac-3 utilizes quantum digits (qdits) with a dimension of 200 discrete modes, allowing for more complex and varied computations.

Qatalyst is a quantum optimization software that allows business experts and programmers to solve complex problems without needing specialized quantum computing knowledge. It provides vendor-neutral access to various quantum processing units (QPUs) and classical processors.

Photonic Quantum Computing and Quantum Key Distribution

This book is about the physical implementation of qubits. However, when dealing with photonic qubits this naturally leads to at least some discussion of quantum key distribution (QKD). The following quote should make this relationship clear: "The QKD network works by transmitting an encoded key in the form of quantum bits (qubits) between endpoints over a fiber optic cable. The qubits are typically polarized photons, which can travel easily along fiber-optic cables. Any attempt to intercept the quantum key destroys the qubit's delicate quantum state and the information it holds, alerting the endpoints that an intrusion occurred. The detectability of the intrusions is what ensures the security of the transmission."[14]

While there are many different QKD protocols, the following steps provide a general outline of how QKD works:

1. The sender transmits photons through a filter (or polarizer) which randomly gives them one of four possible polarizations and bit designations: Vertical (One bit), Horizontal (Zero bit), 45 degrees right (One bit), or 45 degrees left (Zero bit).
2. The photons travel to a receiver, which uses two beam splitters (horizontal/vertical and diagonal) to "read" the polarization of each photon. The receiver does not know which beam splitter to use for each photon and has to guess which one to use.
3. Once the stream of photons has been sent, the receiver tells the sender which beam splitter was used for each of the photons in the sequence they were sent, and the sender compares that information with the sequence of polarizers used to send the key. The photons that were read using the wrong beam splitter are discarded, and the resulting sequence of bits becomes the key.

[14] https://quantumxc.com/blog/what-are-quantum-networks-and-how-do-they-work/

You will see the term "basis" used frequently. Photons in QKD are often in a polarization of 0°, 45°, 90°, or 135°. Measurement generally disturbs a quantum state one can set up a rectilinear measurement or a diagonal measurement. A rectilinear measurement disturbs the states of those diagonal photons having 45/135. As you can see photons are used for quantum key distribution, thus using photonic qubits on either end is quite natural.

A classic QKD protocol is BB84, so named because it was invented by Charles Bennett and Gilles Brassard in 1984. The steps are given here using the ubiquitous Alice and Bob examples:

Step 1: Alice begins with two strings that are n-bits in length. The two strings of bits are encoded as a tensor product of n number of qubits. This will lead to a new vector space.
Step 2: Alice first selects one of two basis states. These are orthogonal or rectilinear. Alice will then encode the two strings (a and b) as a tensor product. Together a_i and b_i provides an induct to four qubit states.
Step 3: Alice then prepares a photon polarization state depending on the bit value and basis state. This is shown in Fig. 3.4.
Step 4: Alice then transmits that photon in the state specified to Bob. This process is then repeated from the random bit stage, with Alice recording the state, basis, and time of each photon sent.
Step 5: Bob then measures the photons in random orientations, as shown in Fig. 3.5.
Step 6: Bob tells Alice in the open what orientations he used, but not what bit values he measured.
Step 7: Alice sends Bob in the open a list of positions at which the orientations are correct.

Again, this chapter is focused on photonic qubits, but this brief description of quantum key distribution should make it quite clear that photonics are used both in qubits and quantum key distribution. This is one advantage of photonic quantum computing.

The BB84 protocol is one good example of a QKD protocol. BB84 was developed in 1984 by Charles Bennet and Gilles Brassard (hence the name). The concept is to encode every bit of the secret key into the polarization state of a single photon.

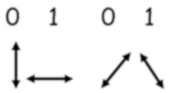


Fig. 3.4 BB84 polarization

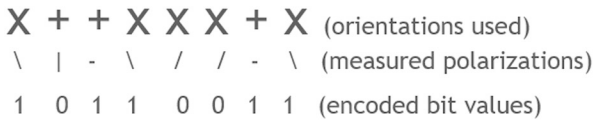


Fig. 3.5 Bob measuring

Because the polarization state of a single photon cannot be measured without altering the photon, this information will be "fragile" and not available to the eavesdropper.

SARG04 is a protocol derived from BB84. When a message is to be sent, the sending party begins with two strings of bits that are each n bits long. These are encoded as a string of n qubits. The name is an acronym of the inventors' names Valerio **S**carani, Antonio **A**cin, Gregoire **R**ibordy, and Nicolas **G**isin. BB84 and SARG04 are just two examples. The goal of this chapter is not to discuss QKD, but since photonics are used, QKD is briefly described.

Cavity Quantum Electrodynamics

While this topic is not, strictly speaking, about photonic qubits, it is quite closely related. Cavity quantum electrodynamics (cavity QED) explores the interaction between light and matter at the most fundamental level using quantum mechanics. It involves confining photons in a highly reflective optical cavity and manipulating the quantum states of these photons as they interact with embedded atoms or artificial atoms such as quantum dots or superconducting circuits. When discussing cavity QED in the context of qubits for quantum computing, the focus is usually on how these interactions can be harnessed to perform quantum operations essential for computation.

Optical cavities are essentially mirrors placed facing each other, trapping photons between them. The quality of an optical cavity is measured by its finesse, which indicates how well it can confine photons. Higher finesse means that photons can bounce back and forth many times before escaping, increasing their interaction time with the matter inside the cavity. The quantum emitters can be actual atoms, ions, quantum dots, or even artificial atoms like superconducting circuits. These emitters absorb and emit photons, engaging in energy exchange processes that are quantized. This interaction forms the basis of qubit manipulation. Photons are trapped within the cavity and act as a medium for qubit interaction. Their wavelike properties allow for coherent superpositions and entanglement, fundamental aspects of quantum computing.

In cavity QED systems, qubits can be represented by the quantum states of the atoms or quantum dots inside the cavity or by the states of the photons themselves. When photons interact with a quantum emitter inside the cavity, they can transfer quantum information to and from the emitter. This interaction is typically characterized by specific quantum phenomena like the Jaynes-Cummings model, which describes how a two-level system (such as a qubit) interacts with a quantized field (the photons).

The Jaynes-Cummings model (JCM) is a fundamental theoretical framework in quantum optics that describes the interaction between a two-level quantum system (like an atom or a quantum dot) and a single mode of the electromagnetic field within a cavity. The two-level system, which can be an atom with two energy

states (ground state and excited state), acts similarly to a qubit in quantum computing. The two states are typically denoted as $|g\rangle$(ground state) and $|e\rangle$ (excited state). The electromagnetic field within the cavity is treated as quantized, meaning that it consists of discrete energy packets or photons. The field is described by a quantum harmonic oscillator, which has quantized energy levels corresponding to the number of photons (0, 1, 2, ...). The cavity confines the electromagnetic field, allowing for a strong interaction between the field and the atom over prolonged periods. The quality of the cavity is crucial, as it determines the lifetime of the photons within the cavity and thus the strength and duration of the light-matter interaction.

The interaction between the atom and the field is described by the Hamiltonian of the system, which in the simplest form of the Jaynes-Cummings model ignores external influences and assumes no dissipation (loss of energy). The Hamiltonian includes terms for the energy of the atom, the energy of the field, and the interaction energy between the atom and the field. The interaction term typically involves operators that raise and lower the energy of the atom and the photon number simultaneously, reflecting the conservation of energy during absorption and emission processes. For higher photon numbers, the Jaynes-Cummings model predicts the phenomena of collapse and revival of the atomic state's oscillations. Initially, the atomic inversion (difference in probability of being in the excited state versus the ground state) shows clear Rabi oscillations, which then dampen (collapse) due to interference among different photon number states. After some time, these oscillations revive, a striking manifestation of quantum interference effects.

By precisely controlling the frequency and intensity of the photons, as well as the properties of the cavity and the emitters, researchers can manipulate the quantum states of the emitters. These manipulations include creating superpositions (where a qubit is in multiple states simultaneously) and entanglement (where the state of one qubit depends on the state of another). The state of the qubits can be read by measuring the properties of the photons emitted from the cavity. For example, the phase or polarization of the outgoing photons can reveal the state of the qubit.

Cavity QED systems can achieve very high levels of control over quantum states, leading to high fidelity quantum operations. By linking multiple cavities or using multiple quantum emitters within a single cavity, it is possible to scale up the system to handle more qubits, although this remains a significant challenge.

Research

There is extensive research involving photonic qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2022 article published in Nature communications[15] titled "High-fidelity photonic quantum logic gate based on near-optimal Rydberg single-photon source" details a significant advancement the field of photonic quantum logic gates. The researchers developed a high-quality Rydberg single-photon source that significantly enhances the fidelity of quantum logic gates. This source provides near-optimal purity and indistinguishability of photons, crucial for reducing errors in quantum gates.

The experiment successfully demonstrated a photonic quantum logic gate (specifically, a controlled-NOT or CNOT gate) with exceptional fidelity. The truth table fidelity reached 99.84%, and the entangling gate fidelity was 99.69%. These are among the highest fidelities reported for such gates. The gate operation utilized photon-photon interference in a free-space setup, facilitated by the high quality of the photons generated. The gate's operation was dependent on the polarization states of the photons, leveraging non-classical correlations to achieve quantum logic operations.

The photonic source used the Rydberg blockade effect, where the strong interactions between Rydberg atoms prevent multiple excitations, thus ensuring the emission of single photons. Photons were generated through a controlled process involving a two-photon transition in cold rubidium (Rb) atoms, which were then used in the CNOT gate protocol. The purity and indistinguishability of the photons were rigorously tested through Hanbury Brown-Twiss and Hong-Ou-Mandel experiments, demonstrating extremely low multi-photon probability and high overlap between consecutive photons.

A 2023 article[16] titled "Quantum entanglement between optical and microwave photonic qubits" focuses on a significant advancement in quantum communication technology, describing a successful experiment in generating and measuring entanglement between optical and microwave photonic qubits. The researchers developed a chip-scale source of entangled optical and microwave photonic qubits integrating a piezo-optomechanical transducer with a superconducting resonator, which is robust under optical illumination. They demonstrated the capability to drive photon-pair generation processes and employ dual-rail encoding intrinsic to their system to prepare entangled states of microwave and optical photons.

The team placed a lower bound on the fidelity of the entangled state by measuring microwave and optical photons in two orthogonal bases, demonstrating the effective entanglement of these disparate types of photonic qubits. This entanglement source can interface directly with telecom wavelength time-bin qubits and GHz frequency superconducting qubits, bridging a crucial gap for quantum communication and computation platforms.

[15] https://www.nature.com/articles/s41467-022-32083-9

[16] https://arxiv.org/abs/2312.13559

Conclusions

Photonic quantum computing is an important area of research in the physical implementation of qubits. Many different organizations are working on photonic based physical qubits with varying levels of success. Photonic quantum computing has several advantages. The advantages include integrating well with existing technology such as fiber optic networks and a low rate of decoherence. Photonic qubits also do not need to be cooled to very low temperatures and are ideal for quantum key distributions. However, there are also disadvantages. Due to photon absorption and scattering, there can be rather high error rates. Scaling photonic qubits can also be difficult.

Chapter 4
Bose-Einstein Condensate

Introduction

Bose-Einstein condensates are an exotic form of matter that presents many interesting possibilities for physical qubit implementation. When a gas of Boson's to close to 0 K (Kelvin) the Bosons will occupy their lowest quantum mechanical state. This causes specific properties to be displayed, properties which are advantageous for quantum computing. Using these properties for information processing is an interesting field of research in the world of physical qubits.

What Is a Bose Einstein Condensate?

As you are undoubtedly aware, matter exists in many states. Even a student in primary school learns of solid, liquid, and gaseous states. However, there are other states of matter. Another state of matter you may also be familiar with is plasma. Plasma is a state of matter that is characterized by the substantial number of charged particles. These charged particles can be ions, or free electrons. The four fundamental states of matter are liquid, gas, solid, and plasma. Plasma is actually quite common and is associated with stars. Plasma is also artificially generated by taking a neutral gas and either subjecting it to a strong electromagnetic field or hearing it. There are other states of matter, but these four are the fundamental states. You are probably at least somewhat familiar with liquid crystals. You may be less familiar with neutron-degenerate matter typically found in neutron stores. Another exotic form of matter is strange matter. This is a type of quark matter that may exist in neutron stars that are approximately 2 to 3 solar masses.

However, this chapter is focused on a state of matter known as a Bose-Einstein condensate. For those readers who are not familiar with particle physics, Bosons are

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_4

particles that carry a force and have a whole number spin. There are four gauge bosons in the standard model that carry specific forces. Bosons can be either composite (made up of quarks) or elementary. Photons are a good example of elementary bosons. Mesons are composite bosons. All subatomic particles can be divided into two classes: Bosons and Fermions. Fermions have 1/2 integer spin (i.e., 1/2, 3/2, etc.) and obey the Pauli exclusion principle. Just like bosons, some fermions are elementary, and some are composite. An electron is a superb example of an elementary fermion. A proton is an equally appropriate example of a composite fermion. More formally, fermions are particles that follow Fermi-Dirac statistics, whereas bosons follow Bose-Einstein statistics.

There are four gauge bosons in the standard model that carry specific forces:

- Photon
- Gluon (there are actually different types of gluons)
- Charged weak bosons (there are two types of these)
- Neutral weak bosons

In addition, there is the Higgs boson, which you have undoubtedly heard of. Guage bosons are simply bosons that carry some fundamental force. As you can see from the preceding list, there is a different gauge boson for each of the fundamental forces.

Just to clarify some of the particle physics involved, a brief overview of the various particles is discussed here. There are three basic types of elementary particles: hadrons, bosons, and fermions. These three groups can be further subdivided. For example, fermions include all quarks and leptons. Leptons are fundamental particles that have a half integer spin. Examples are the electron, muon, and tau. In fact, there are two families of leptons shown in Table 4.1.

Quarks are fundamental particles that are the building blocks of matter in the universe, specifically the constituents of hadrons such as protons and neutrons, which are found in the nuclei of atoms. They were first proposed in 1964 by physicists Murray Gell-Mann and George Zweig, who theorized the existence of quarks to explain patterns seen in the properties of particles discovered in high-energy experiments. There are six types (flavors) of quarks: up, down, charm, strange, top, and bottom. Each flavor has a unique set of properties, including mass, charge, and interactions. Quarks possess a fractional electric charge, either +2/3 or −1/3 times the elementary charge. For example, up, charm, and top quarks have a charge of +2/3, while down, strange, and bottom quarks have a charge of −1/3. Quarks are never found in isolation; they are always confined within hadrons. This is due to a

Table 4.1 Leptons

Charged	Neutral
Electron	Electron neutrino
Muon	Muon neutrino
Tau	Tau neutrino

property known as "color confinement," which dictates that quarks must combine to form composite particles such that the resulting particle is color neutral or "white." Do not be overly alarmed at the term "color." As you can tell from terms like quark, charm, etc., the particle physicists who have described quarks can be a bit whimsical in naming. Color does not refer to visible colors. Rather "color" is related to the strong nuclear force that binds quarks together. Each quark can carry one of three color charges: red, green, or blue.

Hadrons are made up of quarks and/or anti quarks. You are undoubtably familiar with Hadrons. The two most common examples are the proton and neutron. Hadrons can be divided into two groups: Baryons and Mesons. Baryons are composed of three quarks and this group includes the proton and neutron. Mesons are made up of a quark and anti-quark pair. Outside the nucleus of the atom, mesons are rather short lived. This brief interlude of particle physics should provide you with a background for further discussion of Bosons.

Fermi-Dirac statistics, named after Enrico Fermi and Paul Dirac, are one of the two fundamental statistical distributions used to describe the behavior of particles in quantum mechanics. It specifically applies to particles that are indistinguishable fermions, such as electrons, protons, and neutrons, which obey the Pauli exclusion principle. The Pauli exclusion principle states that no two fermions can occupy the same quantum state simultaneously. This principle fundamentally influences the behavior of fermions in a system, particularly in understanding the distribution of particles among different energy states. Fermi-Dirac statistics describe the probability distribution of finding fermions in different energy states at thermal equilibrium, which is a state where the system's temperature remains constant over time. This function gives the probability that a given energy level E is occupied by a fermion at thermal equilibrium. It is defined by the formula shown in Eq. 4.1.

$$f\left(E\right) = \frac{1}{e^{(E-\mu)/kT}} + 1 \tag{4.1}$$

In that formula:

E is the energy of the state.
μ is the chemical potential (i.e., the energy required to add one more particle to the system, and it is a measure of the system's tendency to gain or lose particles).
k is Boltzmann's constant ($\approx 1.380649 \times 10^{-23}$ J/K).
T is temperature.

Einstein statistics, also called Bose-Einstein statistics, describe the statistical behavior of particles that are indistinguishable bosons. In this context, indistinguishability means that two or more bosons cannot be distinguished from one another, and there is no restriction on the number of bosons that can occupy the same quantum state. The primary function for Einstein statistics is the Bose-Einstein distribution function, shown in Eq. 4.2:

$$f(E) = \frac{1}{e^{(E-\mu)/kT}} - 1 \tag{4.2}$$

You may note that this is almost identical to the Fermi-Dirac distribution with the difference being the one in this case is −1, whereas with the Fermi-Dirac distribution it is +1. One of the most remarkable consequences of Einstein statistics is Bose-Einstein condensation, where a macroscopic number of bosons occupy the same quantum state at low temperatures. This phenomenon was predicted by Einstein and Bose in the 1920s and was later observed experimentally in ultra-cold atomic gases. At the temperature where Bose-Einstein condensation occurs, the chemical potential μ approaches zero, indicating that all particles occupy the lowest energy state available.

There are four vector bosons that act as carriers for the four fundamental forces. Photons carry the electromagnetic force, gluons the strong nuclear force, neutral weak bosons (Z bosons) and charged weak bosons (W+) carry the weak nuclear force. In addition, there is the Higgs boson, which you have undoubtedly heard of. There is more detailed information physics relevant to quantum computers, including basic particle physics in Appendix A.

When a gas of Boson's is cooled to such low temperatures (just slightly above 0 Kelvin), a large portion of the Bosons will occupy their lowest quantum mechanical state. In this state, motion is at a minimum. This causes them to being to aggregate and enter the same energy state. Furthermore, in that state, some of the quantum mechanical phenomena, such as wave function interference, become apparent at macroscopic levels. In such super cooled state, essentially the separate particles coalesce into a single quantum mechanical entity. This form of matter was first produced by Albert Einstein based on work done by Satyendra Nath Bose. However, the first actually Bose-Einstein condensate was formed in 1995 by Carl Wieman and Eric Cornell when they cooled a gas of rubidium atoms to 1.7×10^{-7} K. Later that year, Wolfgang Ketterle also produced a Bose Einstein condensate. The three shared the Nobel prize in physics in 2001 for their work. It also happens that this only occurs with particles that have a total spin that is an integer multiple of the reduced Plank's constant $\hbar$ ($6.62607015 \times 10^{-34}$ J/s/2π. The reduced Plank constant is shown in Eq. 4.3.

$$\frac{6.6260701^{5} \times 10^{-34}\,\mathrm{J/s}}{2\pi} \tag{4.3}$$

Both Plank's constant and the reduced Plank's constant are used throughout quantum physics and quantum mechanics. One important issue with Bose Einstein condensates is that a gas at room temperature has atoms (or Bosons) spread across a larger area. The area is given by the thermal de Broglie wavelength shown in Eq. 4.4:

$$\lambda_{\mathrm{dB}} = \left(2\pi h - \mathrm{bar}^{2} / k_{\mathrm{g}} mT\right)^{1/2} \tag{4.4}$$

Note that λ_{dB} is sometimes denoted as $\wedge$. In the preceding equation, kB is the Boltzmann constant, m is the atomic mass and T denotes the temperature of the gas. At room temperatures this de Broglie wavelength is several orders of magnitude smaller than the average spacing between those atoms or particles. Therefore, matter waves for the individual atoms or particles are uncorrelated. In this form, the gas can be described using classical mechanics, more specifically Boltzmann statistics. Whenever the thermal de Broglie wavelength is substantially smaller than the interparticle distance, the gas is classical, often called a Maxwell-Boltzmann gas. As the gas cools the space between the atoms or particles decreases, and at some point (close to 0 K) there is more than one atom or particle in each three-dimensional space of λ_{dB}. At that point the atoms or particles that are adjacent overlap and behave in synchronization. When the interparticle distance becomes substantially smaller than the thermal de Broglie wavelength, quantum effects will dominate and the gas is no longer a Maxwell-Boltzmann gas, but rather a Fermi or gas.

One way to create a Bose-Einstein condensate is to start with a cloud of gas made up of atoms of rubidium. It is then cooled using lasers. The lasers remove energy from the atoms causing them to cool. Once that first stage of cooling is done, the next stage is evaporative cooling. The evaporative cooling allows higher energy particles to escape from the sample, thus reducing the temperature of the gas. The evaporative cooling process is facilitated utilizing electromagnetic fields to isolate the particles from their environment/container.

BEC Qubits

One of the challenges for any physical implementation of qubits is that quantum states are quite sensitive to their surroundings. This leads to the problem of decoherence. Furthermore, quantum states are normally only observable at extremely microscopic levels. However, in Bose Einstein condensates all the bosons occupy the ground state, and therefore the state of the system obeys a macroscopic wave function, the Gross-Pitaevskii equation. One form of the equation is shown in Eq. 4.5.

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{h^2}{2m} \nabla^2 + v_{ext} + g|\psi|^2 \right) \Psi \tag{4.5}$$

In Eq. 4.5:

m is the mass of the particles in the condensate.
$|\psi|^2$ is the atomic density.
V_{ext} is an external potential.
g is a parameter measuring atomic interactions.
∇^2 is the Laplacian, representing kinetic energy.

Another form of the equation is shown in Eq. 4.6:

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t}=\left[-\frac{h^2}{2n}\nabla^2+V(r)+g\left|\Psi(r,t)\right|^2\right] \tag{4.6}$$

In Eq. 4.6:

$\psi(r,t)$ is the wavefunction of the condensate.
$\hbar$ is the reduced Planck constant.
m is the mass of the bosons.
$V(r)$ is the external potential.
G is the interaction strength, proportional to the s-wave scattering length of the bosons. It represents the nonlinear interaction between particles in the condensate and is defined as $g = 4\pi\hbar 2a/m$, where a is the s-wave scattering length.
∇^2 is the Laplacian, representing kinetic energy.

The Gross-Pitaevskii equation is a mean-field theory, where the effects of all other particles on any given particle are averaged, leading to a single-particle wave equation. The term $g|\psi(r,t)|^2$ introduces nonlinearity due to particle interactions. This nonlinearity is critical for many interesting properties of Bose-Einstein condensates, such as the formation of vortices and solitons. The Gross-Pitaevskii equation is a cornerstone in the theoretical study of Bose-Einstein condensates, providing deep insights into the behavior of ultra-cold quantum gases and various exotic quantum phenomena.

If the mathematics of this equation seems a bit daunting, do not be overly concerned. One can certainly proceed with this chapter with only a general, visceral understanding of the Gross-Pitaevskii equation.

The Gross-Pitaevskii equation describes the ground state of a quantum system of identical bosons. The equation utilizes the Hartree-Fock approximation for determining the wave function and energy of a many body quantum system that is in a stationary state (i.e., the Bosons in a ground state). The reason an approximation is necessary is that solving the Schrödinger equation can be quite difficult and is generally only done for single particle systems.

The idea of the Hartree-Fock method dates to the late 1920s after the publication of the Schrödinger equation. In 1927 Hartree introduced a method to calculate approximate wave functions. He referred to this method as the self-consistent method. There were some issues with the Hartree method, particularly it did not account for the principle of antisymmetry of the wave function. The Hartree-Fock method essentially takes the multi-particle wave function and breaks it down into a set of one-particle wave functions.

Computation is accomplished via utilizing two component Bose-Einstein condensates. BECs exhibit quantum superposition and can be entangled. In BEC quantum computing, these phenomena are exploited for computational purposes. Each BEC can represent qubits in a superposition of states. The particles in a BEC behave

as a single quantum entity with a macroscopic wavefunction. This coherence allows for the simultaneous manipulation of a large number of qubits. Quantum bits can be represented by different internal states (such as hyperfine levels) of the atoms in the BEC. Quantum gates are implemented through controlled manipulations using electromagnetic fields.

BECs in optical lattices (periodic potential wells created by intersecting laser beams) can be used for quantum simulation and computation. Each well can trap a single atom, serving as a qubit. The lattice geometry can be adjusted to control interactions between qubits.

Advantages of BECs:

- Scalability: BECs offer a platform where many qubits can be created and manipulated simultaneously.
- Long Coherence Times: BECs can have relatively long coherence times due to their macroscopic nature and isolation from the environment.
- Error Resistance: Certain implementations like topological quantum computing offer intrinsic error resistance.

Challenges and Limitations of BECs:

- Control and Isolation: Precise control over the BEC and its isolation from environmental disturbances are challenging.
- Decoherence: Despite long coherence times, BECs are not immune to decoherence, especially in complex setups.
- Technical Complexity: Creating and maintaining BECs require extremely low temperatures and sophisticated equipment.

However, the primary benefits of using a Bose-Einstein condensate as a qubit is that this approach can reduce the number of qubits needed for a particular application. This is due to the fact that each qubit can be encoded in a collective state of many bosons. Researchers at MIT have proposed to use two BECs of photons as qubits, where each BEC can contain up to 10^5 photons. This way, they can achieve a quantum advantage with only two qubits instead of the numerous (hundreds or even thousands) needed with other approaches.

Gates

Applying quantum gates to Bose-Einstein condensate (BEC) qubits is a complex process that involves manipulating the quantum states of the condensate to perform computational operations. The implementation of gates in a BEC-based quantum computing system relies on precise control over the interactions and external fields affecting the condensate. In the following subsections, common gate implementations used with Bose-Einstein condensate qubits will be described.

Laser Gates

Lasers are particularly useful for controlling the internal and external degrees of freedom of the atoms in a BEC. By adjusting the frequency, phase, and intensity of laser beams, researchers can create potential wells or barriers that precisely manipulate the quantum states of the atoms. This method is often used to induce transitions between different quantum states, effectively implementing quantum logic gates. Lasers are commonly used for gate operations with other physical qubit implementations such as trapped-ion qubits.

Lasers can be used to create potentials and fields that control the internal states and positions of the atoms in a BEC. This manipulation is achieved through various techniques such as optical lattices, which consist of standing waves of laser light creating periodic potential wells where atoms can be trapped. By adjusting the intensity and phase of the lasers, atoms can be moved and controlled within the lattice.

Raman lasers can also be used to induce transitions between different hyperfine states of the atoms, allowing for precise control of the qubit states. Raman lasers are a type of laser that exploit the Raman scattering effect to generate laser light at wavelengths different from those of the pump lasers. Raman scattering involves the inelastic scattering of photons, where the scattered photons have different energies (and therefore different wavelengths) than the incident photons. This change in energy corresponds to the excitation of vibrational, rotational, or other low-frequency modes in the medium.

When light interacts with a material, most photons are elastically scattered (Rayleigh scattering), meaning they have the same energy before and after scattering. However, a small fraction of photons undergoes inelastic scattering (Raman scattering), where they either lose energy (Stokes scattering) or gain energy (anti-Stokes scattering) by exciting or de-exciting molecular vibrations in the medium.

In a Raman laser, a pump laser provides photons at a specific wavelength. Through Raman scattering in a suitable medium (solid, liquid, or gas), some of these photons are converted to photons of lower energy (longer wavelength) corresponding to Stokes scattering. The medium exhibits Raman gain at these new wavelengths, amplifying the scattered light and leading to laser action.

A Raman laser has three primary components. The first is a pump laser, which is a high-power laser that provides the initial photons. The wavelength of the pump laser is chosen based on the Raman-active modes of the medium. The second component is the Raman medium, which is the material in which Raman scattering occurs. Common Raman media include gases (such as hydrogen or deuterium), liquids (such as benzene or water), and solids (such as optical fibers or crystals). The third is the optical resonator which consists of mirrors or other optical components form a cavity that supports the feedback and amplification of the Raman-shifted light, leading to laser emission.

Lasers can also be used to create entanglement between qubits in a BEC by inducing interactions that correlate their quantum states. After performing quantum operations, laser light can be used to measure the states of the qubits. This is

typically done by using fluorescence or absorption imaging, where the response of the atoms to the laser light reveals their quantum state.

Magnetic Field Gates

Quantum gates can be implemented in BEC qubits using external magnetic fields. These fields are carefully calibrated to alter the states of the atoms within the condensate without causing decoherence. For example, magnetic fields can be used to manipulate the spin states of atoms, which are critical for encoding and processing quantum information. Atoms in a BEC can have magnetic moments if they possess a nonzero total spin. These magnetic moments make the atoms sensitive to magnetic fields, which can alter their quantum states. By varying the magnetic field, one can control the orientation and energy levels of these spins.

When an external magnetic field is applied to the Bose-Einstein condensate, magnetic moments of the atoms interact with the field, leading to Zeeman splitting. Zeeman splitting is a quantum mechanical phenomenon where the energy levels of atomic or molecular magnetic dipole moments are split into multiple sub-levels under the influence of an external magnetic field. This effect can be used to selectively address different states. By tuning the magnetic field, specific transitions between these split levels can be induced, effectively performing quantum gate operations such as NOT gates or rotations around the Bloch sphere.

Zeeman splitting occurs because the magnetic field exerts a torque on the magnetic moments of atoms or molecules, aligning them with or against the magnetic field. The energy associated with this alignment depends on the orientation of the magnetic moment relative to the magnetic field direction. Zeeman splitting, also known as the Zeeman effect, refers to the splitting of spectral lines of atoms and molecules when they are subjected to an external magnetic field. This phenomenon is named after the Dutch physicist Pieter Zeeman, who discovered it in 1896.

Gradient magnetic fields have also been used as gates. A gradient magnetic field is a magnetic field that changes linearly in strength over a certain distance. This means that the field strength is not uniform but varies with position. Gradient magnetic fields are typically generated by gradient coils, which are special electromagnets designed to produce a magnetic field that varies linearly along a specific axis (x, y, or z).

Applying a gradient in the magnetic field across the BEC allows for spatially selective addressing of qubits. Atoms at different locations within the condensate experience slightly different magnetic fields, allowing for localized control of their quantum states. This method is particularly useful in systems where the qubits are spatially resolved, enabling individual qubit operations without affecting neighboring qubits, crucial for scaling up quantum computing architectures.

By manipulating the local external magnetic fields that affect only a specific region of the BEC, single-qubit operations such as rotations can be performed. These are fundamental for any quantum computation. Interaction-based gates such as controlled-phase or controlled-NOT gates can be implemented by controlling the

interactions between two spatially separated parts of the BEC. This might involve tuning the barrier or coupling between different wells in an optical lattice to allow controlled interaction when needed.

Figure 4.1 shows one example of a diagram for a Bose Einstein condensate system using magnetic fields.

There are several components in Fig. 4.1 that require some explanation. An Ioffe coil, often referred to in the context of a magnetic trap, is a type of coil arrangement used to create a magnetic field with a specific geometry for trapping and manipulating charged particles, such as ions or neutral atoms with magnetic moments. A quadrupole coil is a specific arrangement of electromagnetic coils designed to produce a quadrupole magnetic field. This type of magnetic field has a gradient that increases linearly with distance from the center in all directions, creating regions with varying magnetic field strengths.

A quadrupole magnetic field is characterized by having four poles—two north poles and two south poles arranged in an alternating pattern. The field strength increases linearly with distance from the center along the radial directions. The primary purpose of quadrupole coils is to produce a magnetic field gradient, which can focus or defocus charged particle beams or manipulate magnetic moments of atoms and molecules. Electromagnetic Quadrupoles use current-carrying coils to generate the magnetic field. They can be adjusted by varying the current. Permanent Magnet Quadrupoles use permanent magnets to produce a static quadrupole field. They are simpler but less adjustable than electromagnetic quadrupoles.

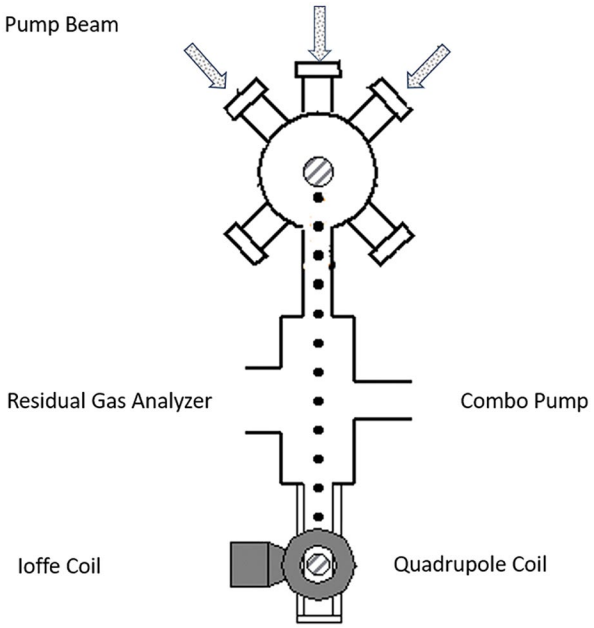


Fig. 4.1 BEC device

A residual gas analyzer is an essential tool used in the study and maintenance of ultra-high vacuum (UHV) systems, which are crucial for experiments involving BECs. An RGA is a type of mass spectrometer designed to detect and analyze trace amounts of gas present in a vacuum system. It identifies the types and partial pressures of residual gases within the vacuum chamber. It works by ionizing gas molecules and then measuring their mass-to-charge ratios, providing a spectrum that indicates the presence and concentration of various gas species.

Spatial Separation Gates

Some approaches use the spatial separation of the condensate into distinct regions to encode and manipulate qubits. Techniques such as optical lattices (arrays of microtraps created by intersecting laser beams) can hold and control multiple BEC qubits simultaneously, allowing for parallel gate operations across a quantum register. Optical lattices are formed by the interference pattern of intersecting laser beams, creating a periodic potential landscape for the atoms in the BEC. These can trap atoms in regular arrays of microscopic potential wells. By selectively addressing individual wells, quantum gates can be applied to specific locations without influencing others. This is particularly useful for operations such as quantum simulation and scalable quantum computing architectures.

Spatial separation gates can use optical tweezers or magnetic fields to manipulate qubits. Focused laser beams can act as optical tweezers to trap and move individual atoms within the BEC. By precisely controlling the position of the optical tweezers, atoms can be spatially separated or brought together. Magnetic field gradients can be used to create potentials that move atoms based on their magnetic moments. This technique allows for the spatial separation of atoms with different internal states

Practical Use-Cases

A study first published in Materials Today Electronics, and reprinted at Phys.org details the use of Bose-Einstein condensates consisting of large numbers of bosons, to create an information processing apparatus. The researchers reported that the Bose-Einstein condensate cloud was taking part in Rabi oscillations indicating that they can be used as exitonic qubits. Not that an exciton is a quasi-particle that forms in a semiconductor or an insulator when an electron is excited from the valence band to the conduction band, leaving behind a positively charged hole. The electron and the hole, attracted to each other by Coulomb forces, pair together to form this bound state. Excitons are key players in the study of optical properties in materials, particularly in the context of absorption, emission of light, and energy transfer processes.

For those readers not familiar with Rabi oscillations, they describe the coherent oscillation of quantum states within a two-level system, such as a spin-1/2 particle or a two-state quantum bit (qubit), under the influence of an external oscillatory field, typically a radiofrequency or microwave magnetic field. Rabi oscillations occur when a two-level quantum system is subjected to a resonant driving force that matches the energy difference between the two levels. A two-level system is a system with two energy states, $|0\rangle$and $|1\rangle$separated by an energy difference ΔE. This system could represent an electron spin in a magnetic field, where $|0\rangle$and $|1\rangle$ correspond to the spin being aligned and anti-aligned with the field, respectively. A driving field, such as a magnetic field oscillating at a frequency close to $\nu = \Delta E/h$ (where h is Planck's constant), is applied to the system. This field is generally perpendicular to any static field present. If the frequency of the external field matches the transition frequency between the two energy states (resonant frequency), the system absorbs energy from the field, leading to transitions between the $|0\rangle$ and $|1\rangle$ states.

The dynamics of Rabi oscillations can be described mathematically using the Rabi formula, which predicts the probability P that a quantum system will be found in a particular state as a function of time t. This is shown in Eq. 4.7:

$$P(t) = \sin^2\left(\frac{\Omega_R t}{2}\right) \tag{4.7}$$

In the preceding equation, Ω_R is the Rabi frequency, whose value will depend on the strength of the driving field and the coupling field. The Rabi frequency provides a determination of how fast the system oscillates between the two states.

Research

There is extensive research involving photonic qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

An article published in Nature Communications Physics,[1] titled "Classical analog of qubit logic based on a magnon Bose-Einstein condensate" explores the potential of using magnon Bose-Einstein condensates (BECs) at room temperature as a classical analog for quantum bit (qubit) functionalities. Magnons (quanta of spin waves in magnets) can form a BEC at room temperature in yttrium-iron-garnet (YIG) ferrimagnetic films. A magnon BEC represents a classical version of qubit functionalities, not requiring ultra-low temperatures like traditional quantum systems. Magnon BECs are formed through parametric pumping, where external microwave photons split into magnon pairs at specific wavevectors, creating a

[1] https://www.nature.com/articles/s42005-022-00970-8

coherent macroscopic quantum state at the lowest energy levels of the system. The magnon BECs exhibit behaviors such as supercurrents, quantized vorticities, and Josephson oscillations, typical of quantum systems but observable at room temperature. The research demonstrates the potential to perform operations akin to those in quantum computing, such as Rabi oscillations (energy exchanges typical in quantum systems), using the magnon BEC setup.

A 2022 article published at Phys.org[2] documents how researchers at the University of Tokyo successfully created the first Bose-Einstein condensate (BEC) from quasiparticles. Quasiparticles, which are not elementary particles but exhibit similar properties like charge and spin, were used to create a BEC. This research shows that quasiparticles can undergo Bose-Einstein condensation, a process previously known to be applicable only to real particles.

The experiment was conducted using cuprous oxide (Cu_2O) where paraexcitons (electron-hole pairs where the spins are anti-parallel) were used due to their long lifetimes and stability compared to orthoexcitons. Paraexcitons were cooled to a few hundred millikelvins using a dilution refrigerator, a device that is crucial in many low-temperature physics experiments and quantum computing. The researchers successfully trapped these excitons in an inhomogeneous strain field within the cuprous oxide, which acted as a potential trap. The exciton BEC was directly visualized in real space using mid-infrared induced absorption imaging. This technique enabled precision measurements of the density and temperature of the excitons, allowing researchers to compare the properties of this quasiparticle BEC with those of traditional atomic BECs.

Conclusions

Bose-Einstein condensate qubits have relatively long coherence times, are error resistant, and can be scaled easily. However, precise control of qubits is difficult and working with BEC is technically complex. There are multiple ways to implement gates including lasers, magnetic fields, and spatial separation. Each of these gate implementations poses its own advantages and disadvantages. BEC qubits have not been explored as fully as photonic, superconducting, and trapped ion qubits, but pose some interesting possibilities making their examination a worthwhile research endeavor.

[2] https://phys.org/news/2022-10-quasiparticle-bose-einstein-condensate.html

Chapter 5
Nitrogen-Vacancy Centers

Introduction

The nitrogen-vacancy (NV) center is an isolated single-spin system trapped in a diamond. Nitrogen-vacancy centers are specific defects that arise during the formation of the diamond. During the formation of the diamond, a nitrogen atom was substituted for a carbon atom creating a vacancy in the carbon lattice. This vacancy can be in two charge states: Neutral and negative. The nitrogen atom on has five valence electrons. Three of t/hem are covalently bonded to the carbon atoms, while the other two remain non-bonded and are called a lone pair. The vacancy on the other side has three unpaired electrons and two of those form a quasi-covalent bond and one remains unpaired. The qubit is stored in the spin of that lone electron. For those readers not familiar with electron spin, electrons have a property called spin, which can be thought of as a type of intrinsic angular momentum. Each electron can be in one of two spin states: spin-up (+1/2) or spin-down (−1/2).

Nitrogen-vacancy centers are atomic-scale defects that occur naturally or can be introduced intentionally in diamond crystals. The nitrogen-vacancy defect consists of a nitrogen atom substituting a carbon atom and a vacancy adjacent to it in the diamond lattice. This defect creates localized electronic states, including a ground state and excited states, which can be manipulated and measured using external fields and techniques. The spin can be fully controlled by microwave radiation and measured by visible light. Furthermore, its resonance frequency is highly sensitive to magnetic field.

For those readers not familiar with the resonance frequency associated with electron spin, more detail is provided. When a material with unpaired electrons is placed in an external magnetic field, the spins of the electrons will align with or against the magnetic field. This creates two energy levels due to the Zeeman effect—one for electrons with magnetic moments aligned with the field (lower energy) and one for those aligned against the field (higher energy). For those readers unfamiliar, The

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_5

Zeeman effect refers to the splitting of a spectral line into several components in the presence of a static magnetic field. This phenomenon is named after the Dutch physicist Pieter Zeeman, who discovered it in 1896. The Zeeman effect is an important tool for studying atomic and molecular structure and for understanding the interactions between light and magnetic fields. It occurs due to the interaction between the magnetic field and the magnetic dipole moment associated with the angular momentum of electrons. This interaction modifies the energy levels of the electron's atomic orbitals.

There are actually two types of Zeeman effect. The first, termed the Normal Zeeman effect occurs when the spectral lines are split into three components (one unshifted line and two symmetrically shifted lines) due to the presence of a magnetic field. The is observed when the total spin of the electrons involved is zero. The second type of Zeeman effect is named, not terribly well in my opinion, the Anomalous Zeeman effect. The Anomalous Zeeman effect involves splitting into more than three components. This is due to the combined effects of both orbital and spin angular momenta. You may be surprised to find that the Anomalous Zeeman effect is actually far more common than the Normal Zeeman effect, thus my earlier comment about it being poorly named.

Electron spin resonance measures the transitions of these electron spins between the lower and upper energy states. For resonance to occur, energy in the form of electromagnetic radiation (usually microwave frequency) must be absorbed by electrons. The frequency of this radiation must match the energy difference between the two spin states, which is proportional to the strength of the external magnetic field. The frequency at which resonance occurs is given by the equation:

$$v = g\mu_B B / h$$

In that equation:

V = resonance frequency.
G = the g-factor which is a constant specific to each type of electron.
μ_B = The Bohr magneton, which is a physical constant that relates the energy of an electron spinning in a magnetic field to the strength of the field.
B = Magnetic field strength.
h = Plank's constant.

The resonance frequency thus depends on the magnetic field strength and properties of the electron. However, for at least some readers, some elements of this equation may still be unclear. The first item that may not be clear to all readers is the Bohr magneton. The Bohr magneton is a physical constant that provides a measure of the magnetic moment of an electron caused by its angular momentum at the atomic scale. It is a key concept in quantum mechanics and magnetic phenomena, particularly in the study of atomic and molecular magnetic properties. It is defined as the magnetic moment associated with an electron due to its orbital or spin angular momentum under the influence of a magnetic field. It is denoted by the symbol μ_B

which we saw in the previous equation. As I am sure you will have already surmised, there is a formula for calculating $\mu_{B:}$

$$\mu_B \frac{e\hbar}{2m_e}$$

In that formula:

e = the elementary charge/the charge of the electron.
$\hbar$ = the reduced Planck's constant which is $\frac{h}{2\pi}$.
m_e = the mass of the electron.

The Bohr magneton represents the magnitude of the magnetic dipole moment that an electron would have if it were either in the lowest possible orbit around the nucleus or if its spin were aligned in a magnetic field. The Bohr magneton is expressed in units of Joules per Tesla (J/T) or equivalently, Amperes per square meter (A/m^2), which are standard units for measuring magnetic moments. The value of the Bohr magneton is approximately 9.274×10^{-24} Joules per Tesla. This small value reflects the inherently weak magnetic effects at the scale of electrons compared to macroscopic magnetic phenomena.

In Depth into Diamond Vacancies

Diamonds are composed of carbon atoms arranged in a specific crystal structure called diamond cubic. Each carbon atom in a diamond is bonded to four neighboring carbon atoms, forming a very strong and stable structure. Figure 5.1 shows a simple carbon lattice.

However, sometimes during the formation process or due to external factors like radiation or high temperature, carbon atoms may be removed from the lattice, leaving behind vacancies. These vacancies can affect the properties of the diamond, such as its mechanical, electrical, and optical properties. Diamond vacancies can be classified based on their location and arrangement within the crystal lattice. For example, a single vacancy where one carbon atom is missing is known as a

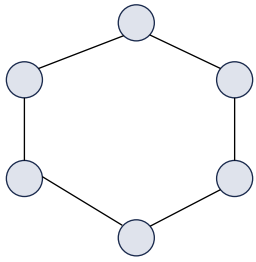


Fig. 5.1 Basic carbon lattice

monovacancy. Similarly, there can be divacancies (two adjacent missing carbon atoms), and other types of vacancies depending on the specific arrangement.

Nitrogen centers in diamonds possess properties similar to single atoms, including long-lived spin quantum states and well-defined optical transitions within a solid-state environment. These make NV centers promising for quantum technology applications. These vacancies can influence the diamond's behavior in various applications. For instance, they can act as charge traps affecting the diamond's electrical properties, or they can alter its optical properties by introducing color centers. Understanding and controlling these defects are important for applications ranging from diamond-based electronics to quantum computing and photonics.

Monovacancy is the simplest type of vacancy where a single carbon atom is missing from the diamond lattice. It leaves behind an unpaired electron, which can affect the diamond's electronic and optical properties. Divacancy is another nitrogen vacancy diamond flaw. In a divacancy, two adjacent carbon atoms are missing from the lattice. This type of vacancy can also introduce unpaired electrons and has been studied extensively for its potential applications in quantum information processing and sensing. Vacancies can also occur in more complex arrangements involving multiple missing carbon atoms. These defects may exhibit unique properties depending on their configuration.

These vacancies can be created artificially. Vacancies can be created in diamond through various processes, including irradiation, high temperature, and chemical treatments. Irradiation with high-energy particles or photons can knock carbon atoms out of the lattice, creating vacancies. High-temperature annealing processes can also lead to the migration and aggregation of vacancies within the diamond crystal.

The vacancy is often filled by an atom other than carbon, such as nitrogen. This is shown in Fig. 5.2.

Figure 5.3 is a three-dimensional image illustrating the nitrogen vacancy.

NV centers have several interesting features that make them useful for quantum computing. Perhaps the most obvious is that electron spin has a very long coherence time. The spin coherence can last up to several and can do so at room temperature. The electron spin in an NV center also couples to nuclear spins, which provides another feature that can be used as qubits that can be used to store and process quantum information.

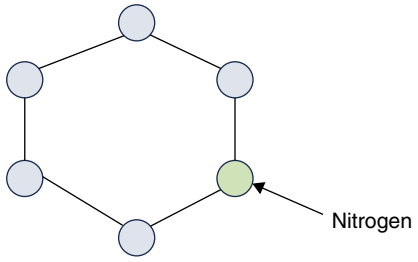


Fig. 5.2 Nitrogen vacancy

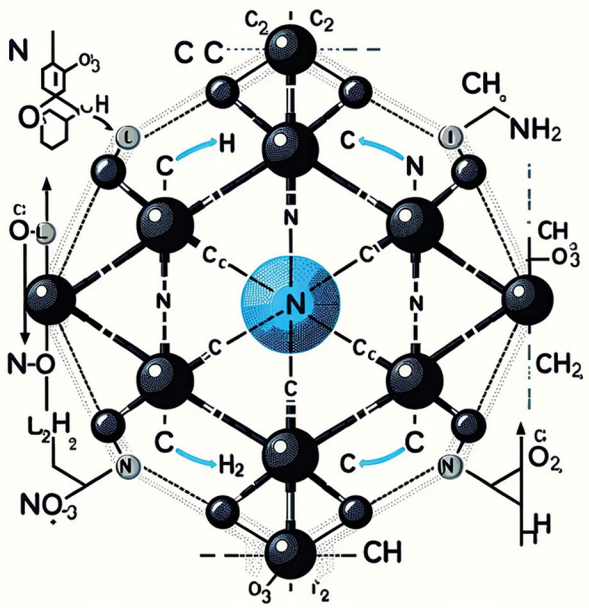


Fig. 5.3 Nitrogen vacancy

As you have already seen in this chapter, and perhaps preceding chapters, delving into physical qubits brings us to various topics in physics. Earlier in this chapter we discussed a bit about electron spins, the Bohr magneton, and the Zeeman effect. We will now look a bit deeper into some related concepts. Let us first consider Landé g-factor. Before delving into its details, allow me to first explain to you why it is relevant. The Landé g-factor is the determining factor in the magnitude of splitting that was previously described in relation to the Anomalous Zeeman effect.

The Landé g-Factor

The Landé g-factor, named after the physicist Alfred Landé. In atomic physics, both the orbital angular momentum and the spin angular momentum of electrons contribute to their overall magnetic moment. The magnetic moment due to orbital angular momentum and the magnetic moment due to spin angular momentum are not generally proportional to these angular momenta by the same factor. This discrepancy led to the introduction of the Landé g-factor to provide a correct scaling factor. As the astute reader will already have surmised, there is an equation related to the Landé g-factor:

$$\vec{\mu} = -g\frac{\mathrm{e}}{2m_c}\vec{J}$$

In the preceding equation:

$\vec{J}$ = the total angular momentum which is the vector sum of the orbital momenta and the spin angular momentum
$\vec{\mu}$ = the magnetic moment
e = elementary charge
m_c = electron mass
g = the Landé g-factor

So now that you have seen how the Landé g-factor is essential in finding the magnetic moment, and you know it is important in the Zeeman effect, the question becomes, how is the Landé g-factor itself calculated. This is actually a rather simple calculation, and is shown here:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

In the preceding equation:

J = the quantum number for the total angular momentum
L = the quantum number for orbital angular momentum
S = the quantum number for the spin angular momentum
g = Landé g-factor

The essence of this interlude into physics is that different values of L, S, and J lead to different g-factors, influencing the degree to which lines are split under the influence of a magnetic field. The Landé g-factor is a fundamental concept in atomic physics and quantum mechanics, linking the abstract quantum numbers of angular momentum to observable physical properties like magnetic moments and spectral lines.

Into the Diamond-Vacancy Qubit

Nitrogen-vacancy (NV) quantum computing is an emerging field that leverages the unique properties of defects in diamond crystals known as nitrogen vacancies to implement quantum computing operations. In NV quantum computing, the electronic spin states of individual nitrogen vacancies serve as qubits, the fundamental units of quantum information. Much of the interest in nitrogen-vacancy center arises from its spin quantum states. In its ground state, the electrons bound to the nitrogen-vacancy center have a net spin S = 1, and superpositions if the spin sublevels (ms =−1, 0, 1) can exhibit coherent quantum evolution over long time scales. The

common sources of disturbance for spins in semiconductors and insulators are spin-orbit coupling and magnetic nuclei in the crystalline structure. These factors are relatively weak for diamond nitrogen centers. For most diamond devices, spin coherence is limited by magnetic interactions with surrounding electronic[4] and nuclear spins.[5] As a result, coherence times for the NV electronic spin range from a few microseconds in diamond with a significant amount of nitrogen impurities (the vast majority of diamonds fall in this category) to milliseconds in ultrapure isotopically purified diamonds. The stability of the spin states in nitrogen-vacancy centers is one reason for the interest in using them in quantum computing. Childress and Hanson[1] state that "Much of the excitement about the NV center stems from its robust spin quantum states. In its ground state, the electrons bound to the NV center have a net spin S = 1, and superpositions of the spin sublevels (ms = −1, 0, 1) can exhibit coherent quantum evolution over long time scales. The typical sources of disturbance for spins in semiconductors and insulators—spin-orbit coupling and magnetic nuclei in the crystalline structure—are relatively weak for NV centers in diamond."

Perhaps the most important advantage to the nitrogen-vacancy qubit is that computation can be carried out at room temperature.[2] Many other approaches to implementing physical qubits require very cold temperatures, well below 0 Celsius. It is also relatively easy to develop stable qubits. Furthermore, the creation of universal quantum gates is relatively straight forward. However, it can be quite difficult to work with a large number of qubits. This final issue has been a limiting factor in the development of quantum computers based on diamond-vacancy qubits.

In nitrogen-vacancy quantum computing, the spin states of electrons associated with the nitrogen-vacancy defect serve as qubits. These electronic spin qubits have long coherence times, which makes them attractive for quantum computation. The ground state of the nitrogen-vacancy defect has an electronic spin-1 system, with three spin states: $|0\rangle$, $|+1\rangle$, and $|-1\rangle$. These spin states can be manipulated using techniques such as microwave and radiofrequency pulses. The qubits are initialized to a well-defined state, typically the $|0\rangle$ state, using optical or microwave techniques.

Quantum gates are applied to the qubits to perform quantum operations. This is typically done using microwave or radiofrequency pulses that drive transitions between the spin states. Quantum gates, such as single-qubit rotations and two-qubit entangling gates, are implemented using external fields and control techniques. These gates are used to perform quantum algorithms and computations. By controlling the duration and frequency of the pulses, arbitrary single-qubit rotations can be implemented, enabling the realization of quantum gates such as the Pauli-X gate, Pauli-Y gate, and Pauli-Z gate. After manipulation, the state of the qubits is measured to extract the results of the computation. Readout can be achieved using techniques such as fluorescence detection or spin resonance. Nitrogen-vacancy

[1] Childress, L., Hanson, R. (2013) DiamondNV centers for quantum computing and quantum networks. Retrieved from https://qutech.nl/wp-content/uploads/2017/03/Diamond-NV-centers-for-quantum-computing-and-quantum-networks.pdf

[2] https://www.nature.com/articles/s41467-021-24494-x

quantum computing has been demonstrated to implement quantum algorithms such as Grover's algorithm, quantum error correction codes, and sensing application.

Two-qubit entangling gates enable interactions between pairs of qubits, creating entanglement between them, which is essential for universal quantum computation. In nitrogen- vacancy quantum computing, two-qubit entangling gates are typically implemented using the interaction between the spin states of neighboring nitrogen vacancy. By controlling the duration and frequency of the pulses, arbitrary single-qubit rotations can be implemented, enabling the realization of quantum gates such as the Pauli-X gate, Pauli-Y gate, and Pauli-Z gate.

One common approach is to exploit the electron-nuclear spin interaction, where the spin of an electron associated with one NV defect interacts with the spin of a nearby nitrogen nucleus, leading to entanglement between the two NV qubits. Another approach is to use the dipolar interaction between two NV defects, which can be controlled using external magnetic fields to realize two-qubit gates. By carefully engineering the interaction between NV qubits and controlling the external fields, two-qubit entangling gates, such as the Controlled-NOT (CNOT) gate, can be implemented.

Besides single-qubit rotations and two-qubit entangling gates, other physical gates can be implemented in NV quantum computing depending on the specific architecture and control techniques. For example, multi-qubit gates, higher-order gates, and non-Clifford gates may be implemented using more complex control sequences and interactions between multiple qubits.

An article in the journal Nature demonstrated the ability to control and measure diamond-vacancy qubits at room temperature.[3] The article also explored the use of nuclear spins in such vacancies as a basis for quantum memory. The authors also state that "The negatively charged nitrogen-vacancy (NV) center in diamond is one of the most attractive solid-state qubit platforms,[18,19,20] for which high fidelity single- and two-qubit NV electron and nuclear qubit gates,[21] as well as quantum error correction protocols,[22] have been demonstrated. Consequently, surrounding ^{13}C (nuclear spin-1/2) and ^{14}N or ^{15}N (nuclear spin 1 or 1/2) atoms can be engineered in the diamond lattice for this purpose, and control of up to 27 nuclear spins via single NV electron spin has been achieved."[23]

Companies Working with Diamond-Vacancy Quantum Computing

While diamond-vacancy quantum computing has not received the attention that photonics, superconducting qubits, or trapped ions have received, there are indeed companies exploring how to create quantum computers with diamond-vacancy qubits. Understanding the approaches these companies are using can help elucidate the entire topic of diamond-vacancy quantum computing.

[3] https://www.nature.com/articles/s41467-021-24494-x#citeas

Quantum Brilliance

Quantum Brilliance is a company founded in 2019 that is working with diamond-vacancy quantum computing.[4] Their product qristal is used for a hybrid C++/Python quantum computing development platform. This can be downloaded from Github.[5] Quantum Brilliance provides an SDK for their simulator allowing anyone to work with the system.

Their system utilizes a bottom-up fabrication technique for diamonds that avoids limitations in surface chemistry and lithography.[6] Their goal is to build quantum accelerators that are over 50 qubits in size. Their current product has 5 qubits and is contained in a 19-inch rack-mountable unit. A picture of that unit is shown in Fig. 5.4.

The goal is to achieve rack mountable units of greater than 50 qubits, then such units can be utilized in racks as massively parallel quantum computing.[7] At the same time, the individual racks are small enough to be deployed for example in robotic autonomous systems.

Diatope

Diatope is another company hoping to bring diamond-vacancy quantum computing to fruition.[8] This company provides diamonds for quantum computing, or a quantum processor. As of this writing they offer a 2 mm × 2 mm or a 4 mm × 4 mm diamond size processor. Perhaps most useful to the reader, their company website discusses what makes a particular diamond appropriate for quantum computing: "The performance of NV centers as (nanoscale) magnetic field sensors in quantum sensing or qubits in quantum computing is deduced from several electro-optical properties of the NV centers. In the rating of NV centers, the criteria always have to be evaluated always in view of the application. NV centers for nanoscale NMR

Fig. 5.4 Quantum Brilliance rack mountable unit

[4] https://quantumbrilliance.com/

[5] https://github.com/qbrilliance/qristal

[6] https://static1.squarespace.com/static/5e54471da3971960f69d9535/t/60583daa7e7c406eeb02b8f2/1664278918857/Whitepaper%2C+March+2021+Edition.pdf

[7] https://quantumbrilliance.com/quantum-technology-application

[8] https://diatope.com/

wide-field magnetic imaging are usually close to the diamond surface and thus suffer from decoherence originating from this (mostly) uncontrollable interface. Here, especially the stability of its negative charge state as well as coherence time are of interest and form indicators for the NV performance and suitability. In the application field of quantum computing, the NV centers are usually deeper in the diamond and already possess long (up to milliseconds) coherence times. Here, the coupling strength and efficacy to other spins (NV centers or 13C nuclear spins) within the diamond matrix[9] are the parameter that determine the performance of NV center as qubit registers."

QZabre

QZabre is a Swiss company[10] that makes various products using nitrogen vacancy centers. These products include quantum scanning microscopes, quantum scanning tips, and diamonds with nitrogen vacancies. This company does not produce actual quantum computers but does produce equipment that is needed for creating quantum computers. The existence of companies that are suppliers for quantum computing vendors is an indicator that the quantum computing industry is maturing.

QZabre combines NV measurements with scanning probe microscopy, providing a user-friendly, turnkey system. This system allows for high-resolution imaging of magnetic fields at the nanoscale, with applications in materials science, quantum physics, and spintronics. Their probes are equipped with integrated microwave antennas, which simplifies the setup process and enhances performance by ensuring optimal alignment and minimizing the risk of damaging the scanning tips.

QZabre offers a range of NV diamond probes, from single NV centers to ensemble NV tips, tailored for various experimental needs. These probes are known for their high optical quality, stability, and ease of integration with existing atomic force microscopy (AFM) systems. The latest quality grade Q8 sensors provide magnetic sensitivities better than 1.5 $\mu T/\sqrt{Hz}$, enabling faster scan speeds and reduced noise in the data.

Research

There are several papers published in the past few years that provide insight into advances in nitrogen vacancy center qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

[9] https://diatope.com/faqs

[10] https://qzabre.com/en

A 2022 paper[11] titled "High Fidelity Control of a Nitrogen-Vacancy Spin Qubit at Room Temperature using the SMART Protocol" explores advanced methods to enhance the control and coherence of nitrogen-vacancy (NV) centers in diamond. The study introduces the SMART (Sinusoidally Modulated, Always Rotating and Tailored) protocol, an advanced form of qubit control that improves upon traditional dynamical decoupling and dressed state methods. This protocol uses continuous microwave modulation to protect the qubit from decoherence and environmental noise, significantly enhancing qubit performance. The SMART protocol extended the coherence times of NV spin qubits by approximately 30 times compared to unmodulated (bare) qubits. This was achieved by reducing the effects of magnetic field fluctuations and spin bath decoherence.

The experiments in "High Fidelity Control of a Nitrogen-Vacancy Spin Qubit at Room Temperature using the SMART Protocol" were conducted using a negatively charged NV center in a chemically vapor-deposited diamond. This setup demonstrated the practical application of the SMART protocol in a standard laboratory environment. The implementation of the SMART protocol resulted in Clifford gate fidelities exceeding 0.99, which is compatible with fault-tolerant quantum computing requirements. This represents a significant improvement over the bare qubit, where fidelity was initially around 0.94.

A 2023 paper published in the Frontiers in Quantum Science and Technology,[12] titled "NV-centers in SiC: A Solution for Quantum Computing Technology?" reviews the advantages of using nitrogen-vacancy (NV) centers in silicon carbide (SiC) for quantum computing, comparing them with NV centers in diamond and divacancy centers in SiC. The article discusses how NV centers and divacancies in silicon carbide present an appealing alternative to traditional qubits used in quantum computing due to their compatibility with current microelectronic technologies and their isolation from environmental disturbances.

While NV centers in diamond are well-studied and show promise, NV centers in SiC may offer advantages due to the material's compatibility with existing semiconductor technologies. The optical manipulation and detection of these centers shift from the visible spectrum (for diamond) to the near infrared (for SiC), aligning with telecommunications wavelengths. The NV centers in SiC have similar spin properties to those in diamond but can be better integrated with silicon technologies, particularly the 3C polytype of SiC, which can be epitaxially grown on silicon. This makes them highly advantageous for scalable quantum computing platforms. SiC-based NV centers address some of the drawbacks of diamond-based systems, such as the need for high-energy lasers for ionization and manipulation. They also offer improved environmental isolation, which is crucial for maintaining the coherence of qubit operations.

[11] https://arxiv.org/abs/2208.14671

[12] https://www.frontiersin.org/articles/10.3389/frqst.2023.1115039/full

Disadvantages and Challenges with Nitrogen-Vacancy Qubits

As you will have no doubt surmised, there are some challenges when working with nitrogen-vacancy qubits. One reason there are so many different approaches to physical qubits is that each has its own advantages and disadvantages. Efficiently initializing and reading out the state of nitrogen-vacancy center qubits can be technically challenging. The initialization process often requires optical pumping, which might not always result in a well-defined spin state. Readout efficiency is also less than ideal, which can lead to errors in quantum computations and measurements.

One of the biggest challenges with nitrogen-vacancy centers is scalability. Creating and maintaining a large number of nitrogen-vacancy centers with uniform properties and reliably coupling them in a controlled way for large-scale quantum computations is a significant technical hurdle. The precise placement of nitrogen-vacancy centers at predetermined positions in the diamond lattice remains a challenge. A somewhat more technical issue is that nitrogen-vacancy centers in diamond can experience inhomogeneous broadening due to variations in the local electromagnetic environment across different nitrogen-vacancy centers. This results in variations in the resonance frequencies of different nitrogen-vacancy centers, complicating the coherent control over multiple qubits.

Creating nitrogen-vacancy centers involves introducing nitrogen impurities into diamond and then creating vacancies nearby, which are subsequently allowed to pair through annealing processes. This process can be difficult to control with precision, leading to variability in the properties of the resulting NV centers. Control of NV center qubits typically requires both optical and microwave fields. Aligning, focusing, and maintaining the stability of these fields over the operation time can introduce complexities and potential sources of error.

Conclusions

As you have seen in this chapter, nitrogen-vacancy centers in diamonds can be used to create qubits that operate at room temperature. In this chapter, the details of these physical qubits have been described along with strengths and weaknesses. Some of the foundational physics underpinning the nitrogen-vacancy center have also been explored. This includes the Zeeman effect, and the Landé g-factor. After studying this chapter, you should have a strong understating of nitrogen-vacancy centers as physical qubits.

Chapter 6
Nuclear Magnetic Resonance Quantum Computing

Introduction

Nuclear magnetic resonance quantum computing is a very interesting approach to physically implementing quantum computers. This approach uses spin states of nuclei within molecules to represent the qubits. The states are probed using nuclear magnetic resonances, thus the name. This system is fundamentally a variation on nuclear magnetic resonance spectroscopy.

Using nuclear spins for quantum computing was first proposed by Seth Lloyd and David DiVincenzo. Seth Lloyd is a professor at MIT who is known for his work in quantum computation. David DiVincenzo is a physicist also known for his work in quantum computing, particularly the well-known DiVincenzo criteria for a quantum computer:

- A scalable physical system with well-characterized qubits.
- The ability to initialize the state of the qubits to a simple fiducial state, such as to $|000\cdots\rangle$.
- A set of universal quantum gates.
- A measurement capability for the qubits.
- Long relevant decoherence times. Specifically, longer than the gate-operation time.

In 2001, IBM was able to implement Shor's algorithm on a 7-qubit NRMQC.

Nuclear Magnetic Resonance

Nuclear Magnetic Resonance exploits the magnetic properties of certain atomic nuclei, specifically nuclei that possess a spin quantum number greater than zero, giving them a magnetic moment. When placed in a strong external magnetic field,

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_6

these magnetic nuclei align themselves in one of two (or more, depending on the spin quantum number) orientations with respect to the field. One of these orientations is with the magnetic field (lower energy state), and one is against it (higher energy state). To measure an NMR spectrum, a radiofrequency (RF) pulse that matches the energy difference between the aligned states (resonant frequency) is applied. This pulse excites the nuclei from the lower energy state to the higher energy state. After the RF pulse is turned off, the nuclei return to their lower energy state, a process known as relaxation. During relaxation, the nuclei emit RF signals, which are detected and recorded. The frequency and intensity of these signals provide detailed information about the molecular environment of the nuclei.

The exact resonant frequency of a nucleus in a molecule depends on the electronic environment around it. This shift in frequency is referred to as the chemical shift, and it is extremely sensitive to the electronic configurations, bonding, and spatial orientation of atoms within the molecule. Nuclei in a molecule can interact with each other through what is called J-coupling or scalar coupling. This interaction can split the NMR signals into multiplets, depending on the number and arrangement of neighboring nuclei. This splitting provides information about the number of neighboring hydrogen atoms (or other relevant nuclei) and their distance relative to the nucleus being observed.

Nuclear Magnetic Resonance Qubits

Nuclear magnetic resonance qubits use the spin states of atomic nuclei as quantum bits. Nuclei, like protons or neutrons, exhibit a quantum property called "spin," which can be thought of as a type of angular momentum. When these nuclei are placed in a magnetic field, their spins align in one of two directions: parallel or antiparallel to the field. These orientations correspond to the two states (0 and 1) of a classical bit. The spin states, typically denoted as "spin up" and "spin down," serve as the 0 and 1 states of qubits. By applying a magnetic field and radio frequency pulses, these spins can be manipulated and put into superposition or entangled states.

Nuclear magnetic resonance quantum computing often uses an ensemble of many molecules, each contributing many identical qubits, to amplify the signal and improve measurement reliability. NMR qubits typically have relatively long coherence times compared to other types of qubits, making them useful for certain types of quantum algorithms. Nuclear Magnetic Resonance quantum computing has been used for several proof-of-concept quantum algorithms. However, it faces scalability issues. Increasing the number of qubits significantly complicates the spectral lines and control of the system, limiting the practical size of an NMR quantum computer. Depending on the specific nucleus being used, each will have different energy levels and therefore different interaction with its neighboring nucleus. This allows us to treat these nuclear as distinguishable qubits.

Liquid State NMR

There are two approaches to this process. These two states are liquid and solid. This approach leverages the magnetic properties of atomic nuclei in a solution to perform quantum computations. In liquid-state NMR, the qubits are based on the spin states of atomic nuclei such as ^{1}H (hydrogen), ^{13}C (carbon-13), or ^{31}P (phosphorus-31), which have a spin of 1/2. These isotopes are favorable because they provide a clear two-level quantum system, essential for defining qubits. In liquid-state NMR quantum computing, a sample containing molecules with the chosen isotopic nuclei is dissolved in a suitable solvent, often deuterated to minimize interference from hydrogen in the solvent. When placed in a strong magnetic field, the nuclear spins align either with or against the magnetic field. This alignment creates two distinct energy states (spin-up and spin-down), which represent the 0 and 1 states of a qubit. RF pulses are used to manipulate the spin states. The frequency, phase, and duration of these pulses are carefully controlled to execute quantum gates. These gates can change the state of a single qubit or entangle multiple qubits, facilitating quantum operations. The quantum state of the nuclei is determined by the RF signals they emit as they relax back to their equilibrium state after excitation by the RF pulses. The emitted signals are detected and can be analyzed to infer the quantum state of the system, which allows the results of computations to be read out.

Liquid state nuclear magnetic resonance qubits have some specific advantages and disadvantages. Nuclei in liquid solutions typically exhibit longer coherence times than electrons in solid-state systems. This is due in part to the fact that molecular tumbling averages out many dephasing interactions. Unlike other quantum systems that use individual particles as qubits, NMR usually involves ensemble measurements where the behavior of large numbers of identical molecules is averaged. This can enhance signal strength but reduces the ability to perform certain quantum algorithms that require individual qubit operations. Another issue is scaling. Scaling liquid-state NMR systems to a large number of qubits is challenging.

Solid State NMR

The second approach is solid state nuclear magnetic resonance quantum computing. This approach takes advantage of the static and well-ordered nature of solids to potentially achieve more scalable and coherent quantum computing architectures compared to liquid-state NMR. Solid-state NMR uses the magnetic properties of nuclei within solid materials. Similar to liquid-state NMR, nuclei that possess spin, such as ^{1}H, ^{13}C, and ^{31}P, are aligned in a strong magnetic field. However, in solids, the magnetic interactions are not averaged out by molecular tumbling as in liquids, which results in broader spectral lines and more complex interactions between nuclear spins.

For solid state nuclear magnetic resonance qubits, it is common to have crystalline samples, while other solid states can be used. The choice of material and its isotopic purity can significantly influence the coherence times and interaction strengths of the qubits. In a strong magnetic field, nuclear spins align either parallel or anti-parallel to the field, creating distinct energy states used to define qubit states (0 and 1). Polarization can be enhanced using techniques like dynamic nuclear polarization (DNP) to improve signal strength. The fixed positions of atoms in a solid lead to strong and consistent spin-spin interactions, which can be used to create entanglement between qubits.

The decoherence times in solids can be longer due to reduced motion, but they are also affected by dipolar interactions and electric quadrupole interactions (in nuclei with spin greater than 1/2). Implementing quantum gates in solid-state NMR can be challenging due to the complexity of interactions and the difficulty in isolating individual qubits. However, advances in nanofabrication and isotopic engineering hold promise for overcoming these challenges.

As with any approach to physical qubits, sold state nuclear magnetic resonance has advantages and disadvantages. The lack of motion in solids leads to spectral crowding, where many lines appear close together, making it difficult to resolve individual transitions necessary for precise qubit manipulation. While solid-state systems potentially offer better scalability than liquid-state systems, managing and controlling an increasing number of qubits within a solid matrix remains a significant technical challenge. The choice of solid-state material and its isotopic enrichment are crucial, as impurities and isotopic disorder can lead to faster decoherence and reduced fidelity of quantum operations.

Kane Quantum Computer

The Kane quantum computer is a theoretical and experimental approach to quantum computing proposed by Bruce Kane in 1998. This model is specifically designed around the use of phosphorus donor atoms embedded in a silicon substrate to create qubits, leveraging both nuclear and electron spins. This approach combines principles from nuclear magnetic resonance (NMR) and more traditional semiconductor technology. In the Kane model, each qubit consists of a phosphorus atom that has been embedded in a pure silicon crystal. The phosphorus atom replaces a silicon atom in the lattice, which creates an “extra” electron (since phosphorus has one more valence electron than silicon). The nuclear spin of the phosphorus nucleus and the spin of this extra electron are used to encode quantum information.

Qubits in the Kane model are initialized to a known state using electromagnetic pulses, typically in the radiofrequency or microwave bands, to manipulate the spin states of the electrons and nuclei. Quantum gates, which are the basic operations that manipulate qubits, are implemented using controlled electromagnetic pulses

that influence the spin interactions between the electrons and the phosphorus nuclei. These gates can create superpositions and entangled states, which are critical for quantum computation. The quantum state of a qubit is read by measuring the spin state of the electron or the nucleus. This process might involve techniques similar to those used in magnetic resonance imaging (MRI) or other forms of spin resonance technology.

One of the key advantages of the Kane model is that both electron and nuclear spins in silicon have relatively long coherence times, particularly when the silicon is isotopically purified to remove any magnetic isotopes that might cause decoherence. The Kane quantum computer was designed with scalability in mind. The original proposal included a plan for an array of donor atoms spaced at regular intervals within a silicon substrate, each atom individually addressable by a set of nearby electrodes. These electrodes would control the electrostatic potential at each donor site, allowing individual manipulation of qubits and their interactions, an essential feature for building a practical quantum computer.

Implementing a Kane quantum computer in practice is quite challenging. It requires extremely precise placement of phosphorus atoms in silicon, very low operating temperatures, and sophisticated techniques to control and measure the quantum states of individual nuclear and electron spins. Despite these challenges, progress has been made in technologies relevant to the Kane model, particularly in the areas of quantum dot and silicon-based quantum computing

Specific Implementations

While practical implementations of nuclear magnetic resonance quantum computers are not as common as other approaches, such as trapped ion, superconducting, and photonic, there are indeed actual quantum computers that utilize NMR for realizing qubits.

SpinQ

The company SpinQ has produced a 2-qubit desktop quantum computer that uses nuclear magnetic resonance.[1] They advertise this as a “desktop quantum computer.” The product is primarily for educational purposes. There specification sheet provides some insight into their product:[2]

[1] https://www.spinquanta.com/products-solutions/gemini

[2] https://www.spinquanta.com/technicalSpec/gemini

Number of qubits	2
Coherence time	≥200 ms
Number of single-bit quantum gate operations	≥200 Gates
Number of multi-bit quantum gate operations	≥20 Gates
Dimensions	Height 600 mm/23.62 inches Width 280 mm/11.023 inches Length 530 mm/20.866 inches Weight 44 Kg/97 lbs

Fig. 6.1 SpinQ Gemini

The SpinQ is shown in Fig. 6.1.

SpinQ also makes variations such as the Gemini-Mini and the Gemini-Lab. There is also a cloud-based variation of SpinQ.[3]

Research

There are several papers published in the past few years that provide insight into advances in nuclear magnetic resonance qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2022 paper[4] entitled "A New Approach to Quantum Computing Multi-Qubit Generation and Development of Quantum Computing Platform with Magnetic Resonance Imaging Techniques" introduces a novel method for generating multiple quantum bits (qubits) using magnetic resonance imaging (MRI). The paper proposes using MRI techniques to generate and manipulate qubits through a concept referred to as MRI gradient-based qubit generation. This utilizes different radiofrequency (RF) coils: one main coil for exciting all qubits simultaneously and multiple

[3] https://cloud.spinq.cn/

[4] https://arxiv.org/abs/2206.05932

smaller Q-coils for individual qubits. The qubits are controlled and manipulated through a process called time-encoded probability amplitudes. This method allows for precise control of qubit states through time-encoded spin-echo sequences, which involve the manipulation of spin states using echo signals to encode the qubits' probability amplitudes.

The method described in "A New Approach to Quantum Computing Multi-Qubit Generation and Development of Quantum Computing Platform with Magnetic Resonance Imaging Techniques" allows for the simultaneous generation of multiple qubits using established MRI technology. This allows for scalable quantum computing. MRI based qubits can also operate and substantially higher temperatures than some other physical qubits. The approach can be integrated with existing MRI systems, making it a potentially cost-effective and less technically challenging option.

A 2024 paper describes how nuclear magnetic resonance, despite being a powerful tool for quantum experiments, faces significant limitations for scaling up to larger quantum computers. This paper is different from others cited in this book. Rather than demonstrate a particular improvement in quantum computing, it focuses on challenges for a specific approach to physical qubits. The scaling issue stems from the difficulty in preparing pure quantum states and managing the tiny energy gaps between nuclear spin levels, which complicates the construction of larger, practical quantum system. The explores advanced quantum control strategies such as the design of composite pulses, shaped pulses, and pulse sequences. These are essential for performing specific transformations of quantum states and improving the control over quantum operations. Table 1 from the article describes the 5 DiVincenzo criteria and how they could be met in NMR systems. That table is shown in Fig. 6.2.

The article suggests that while NMR may not be suitable for building large-scale quantum computers, it continues to be invaluable for proof-of-principle demonstrations and developing techniques that may enhance other quantum computing technologies.

A 2022 article in Nature communications[5] discussed assembly and control of a register consisting of nuclear spin qubits. Nuclear spin qubits are closely related to nuclear magnetic resonance qubits. The researchers introduce a quantum computing platform named Phoenix, which uses the nuclear spin states of strontium-87 atoms as qubits. These atoms are trapped in an array of optical tweezers, allowing for individual manipulation and readout of each qubit. The qubits are trapped using a holographically generated array of optical tweezers. Optical tweezers were discussed in Chap. 3. This setup allows for flexible and dynamic control of the trap geometry, crucial for various computational tasks. One of the significant achievements highlighted in this study was the demonstration of minute-scale coherence times for these qubits, despite the challenge of maintaining quantum coherence over long periods. This is achieved through precise control of environmental interactions and the use of advanced techniques like dynamical decoupling.

[5] https://www.nature.com/articles/s41467-022-29977-z

Criterion	NMR implementation
1. qubits	spin-$\frac{1}{2}$ nuclei
2. initialisation	pseudo-pure states
3. low decoherence	long T_2
4. logic gates	pulses and delays
5. measurement	NMR spectrum

Fig. 6.2 DiVincenzo criteria and NMR

The Phoenix system can individually address each qubit within the register using tailored optical pulses. This capability is essential for scaling up quantum computing operations and performing complex quantum algorithms. The paper delves into the technical aspects of operating the Phoenix system, including atom loading, cooling, state preparation, and error mitigation strategies. It also outlines future directions aimed at increasing the number of qubits and improving gate operations to enhance the system's computational power.

Conclusions

Nuclear magnetic resonance is less common than some of the other physical qubit implementations we have explored in this book. And, as was discussed in this chapter, there are definitely challenges to implementing NMR qubits. However, the possible benefits make this a promising area of quantum research and current publications indicate that there are still many researchers working in this field.

Chapter 7
Electron-Based Quantum Computing

Introduction

Electrons have a property called spin, which can be thought of as a type of angular momentum at the quantum level. Spin can take one of two possible orientations relative to an external magnetic field: "up" or "down" (or +1/2 and −1/2 in quantum mechanical terms). These two states can be used analogous to the binary states 0 and 1 in classical computing. Electron spin states can be manipulated using magnetic and electric fields or through precisely timed microwave pulses. These techniques allow the spin states to be controlled and changed as required for quantum computations. In materials like silicon, phosphorus atoms can be implanted to serve as donor electron spin qubits. The electron from the phosphorus atom can be used as a qubit, with the advantage of a well-understood and extremely pure environment, which helps in maintaining coherence.

Variations

There are several ways that electrons can be utilized to create qubits. Each of these has its own advantages and disadvantages and will be explored in this section.

Free Electron Spin

There are variations of using electrons to store qubits. In 1997 David DiVincenzo and Daniel Loss proposed a type of quantum computer that uses the spin of freedom of electrons, which are confined to quantum dots, to physically implement a qubit.

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_7

This is now called a Loss-DiVincenzo quantum computer. This the Loss-DiVincenzo uses electrons confined to a three-dimensional nano device, to implement qubits.

Electron on Helium

In this approach the basis states are defined by spin states or motional states of an electron trapped on the surface of liquid helium. As early as 1966, the binding of electrons to the surface of liquid helium was demonstrated. In electron on helium systems, electrons are suspended above the surface of liquid helium. The electrons do not sink into the helium because of the Pauli exclusion principle, which prevents them from penetrating into the filled electron shells of the helium atoms. This setup naturally isolates the electrons from many environmental interactions that cause decoherence in other qubit systems. The qubits in this system are based on the quantum states of these floating electrons. These include their vertical motion (above the helium surface) and lateral movement. For quantum computing purposes, these motions can be finely controlled and manipulated to represent and process quantum information.

To manipulate these electron qubits, researchers use microwave and radio frequency electromagnetic fields. By applying these fields, they can control the movement of electrons, inducing transitions between different quantum states. This is analogous to how electrons are manipulated in other quantum systems, like trapped ions or quantum dots. One of the most significant advantages of electron on helium qubits is their exceptionally long coherence times. Electrons on helium are very well isolated from their environment, which minimizes decoherence—a major challenge in quantum computing. The system potentially allows for scaling up. Theoretically, large arrays of these electrons can be created and controlled on the helium surface, enabling the development of larger quantum systems. Electron on helium systems require very low temperatures, typically only a few millikelvins above absolute zero. Maintaining such temperatures is technically challenging and expensive. Manipulating and measuring the state of individual electrons on a helium surface with high precision involves complex setups and fine-tuned control systems.

Silicon Phosphorous

Silicon phosphorous provides a basis for electron spin qubits. Silicon phosphorous qubits represent a cutting-edge approach in the field of quantum computing, utilizing the properties of silicon and phosphorous atoms to store and process information at the quantum level. Silicon has the advantage of already being widely used in the semiconductor industry, which is advantageous for integrating quantum computing technologies with existing fabrication techniques. Moreover, silicon is

known for creating a stable environment for qubits, which helps in maintaining quantum coherence (the time during which a qubit can maintain its quantum state). To enhance coherence times, silicon used in these qubits is often isotopically purified. Natural silicon contains isotopes that can create magnetic noise and disturb the electron spins. Using silicon composed predominantly of the non-magnetic isotope ^{28}Si reduces this noise.

The electron spins are controlled using microwave magnetic fields, and their quantum state can be read out by measuring the spin-dependent properties of the electrons. This manipulation and measurement are critical for performing quantum computations. Silicon Phosphorous qubits are also quite scalable and readily integrated into other systems.

Molecular Magnet Qubits

These types of qubits are not strictly speaking, electron-based. However, they are related. Similar to electron spin qubits, molecule magnet qubits also exploit the quantum mechanical spin states. However, in this case, the spins come from the entire molecule rather than from individual electrons. These molecules can exhibit complex spin ground states due to interactions between the magnetic moments of the atoms within the molecule. Molecule magnet qubits are an intriguing and relatively niche type of qubit utilized in quantum computing, which rely on the magnetic properties of molecules. These molecules are engineered to have specific magnetic characteristics that make them suitable for storing and manipulating quantum information.

The fundamental basis for molecule magnet qubits is the use of single-molecule magnets (SMMs) or polyatomic molecule magnets, which are molecules designed to function like tiny magnets. These molecules can maintain a magnetic moment at very low temperatures, which is essential for their role in quantum computing. Manipulating the spin state of molecule magnets typically involves using external magnetic fields, microwave radiation, or electric fields. Measurements are often carried out through techniques that detect the magnetic or electronic properties of the molecule, such as magnetic resonance methods.

Molecule magnets can have very stable magnetic states, which can potentially maintain coherence for longer periods compared to other types of qubits. The chemical properties of these molecules can be chemically tailored during synthesis, allowing for custom design of the molecule's magnetic properties to suit specific quantum computing needs. Keeping molecule magnets isolated from environmental influences that can cause decoherence (like temperature fluctuations and stray electromagnetic fields) is challenging. Precisely controlling the quantum states of molecule magnets and scaling up the number of qubits in a system remain significant hurdles. Achieving uniformity and consistency across many molecules in a quantum computing system is also difficult.

Frozen Neon

Frozen neon refers to the solid state of the chemical element neon (Ne) at low temperatures. Neon is a noble gas and is normally found in its gaseous state at room temperature. However, when cooled to extremely low temperatures, typically below its boiling point of −27.0389 K (−246.1 °C/−411 °F), becomes a solid. In its solid form, frozen neon appears as a colorless crystalline solid. It retains many of the properties that characterize neon, such as its inertness and lack of reactivity with other elements. Frozen neon is notable for its ability to emit a bright orange-red glow when subjected to electrical discharge, a characteristic shared with gaseous neon. This property makes solid neon useful in certain types of lighting applications, particularly in neon signs and lighting devices.

Frozen neon is not commonly encountered outside of laboratory settings due to the extreme conditions required to maintain it in its solid state. However, its unique optical properties and chemical inertness make it an intriguing subject for scientific research and applications in fields such as physics, materials science, and optics. Quantum computing with frozen neon involves using neon atoms as qubits. In the context of frozen neon, researchers are exploring the possibility of using the spin states of neon atoms as qubits. Spin is an intrinsic property of elementary particles such as electrons and nuclei, and it can be manipulated to encode quantum information. Neon atoms have relatively simple electronic structures, making them promising candidates for qubits. The Ne electron configuration will be $1s^2 2s^2 2p^6$. The structure of Neon is shown in Fig. 7.1.

To use frozen neon for quantum computing, researchers would need to trap individual neon atoms at very low temperatures, typically close to absolute zero, to minimize thermal noise and maintain coherence—the stability of quantum states. Techniques such as laser cooling and magnetic trapping could be employed to

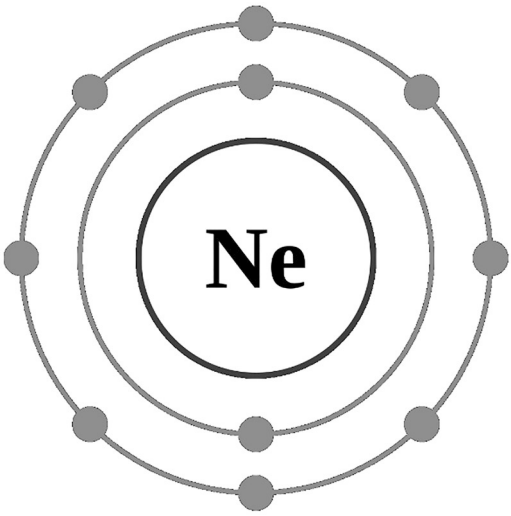


Fig. 7.1 Neon

achieve this. Once the neon atoms are trapped and cooled, one can manipulate their spin states using carefully controlled electromagnetic fields. By encoding information in the spin states of these neon qubits and applying quantum gates via controlled electromagnetic fields, quantum algorithms could be executed to solve certain problems exponentially faster than classical computers.

Deeper into Frozen Neon Qubits

One potential advantage of using frozen neon for quantum computing is its simplicity compared to other qubit implementations, such as trapped ions or superconducting circuits. Neon atoms are relatively easy to manipulate and control, which could simplify the construction and operation of quantum computers. However, quantum computing with frozen neon is still in the theoretical and experimental stages. Significant challenges remain, including achieving long coherence times for the qubits, scaling up the system to larger numbers of qubits, and implementing error correction to mitigate the effects of noise and decoherence. Overall, while quantum computing with frozen neon holds promise for the future of quantum technology, much more research and development are needed to realize its full potential.

Neon atoms would first need to be isolated and cooled to extremely low temperatures, typically close to absolute zero (−273.15 °C or 0 K). This could be achieved using techniques such as laser cooling or magnetic trapping. Once the neon atoms are trapped and cooled, quantum information would need to be encoded in their quantum states. In the case of neon, researchers might consider using the spin states of individual neon Finally, the quantum state of the neon qubits would need to be measured to extract the result of the quantum computation. This measurement process would likely require sophisticated techniques to avoid disturbing the quantum state and to achieve high-fidelity readout. The U.S. Department of Energy Argonne National Laboratory recently (May 2022) announced it had created qubits by freezing neon gas into a solid, and spraying electrons onto the solid. The electrons were then trapped, allowing it to be used as a qubit.[1]

Electrodes in the microchip can keep electrons that get trapped on the solid neon in place for more than 2 months. A superconducting microwave resonator on the chip, much like a microscopic version of a microwave oven, then emits microwaves to help control and read the qubit. Qubits stay in superposition for 220 ns and change state in only a few nanoseconds Electrons that get trapped on the solid neon can be kept in place for more than 2 months.[2]

[1] https://www.sciencedaily.com/releases/2022/05/220504130823.htm

[2] https://spectrum.ieee.org/neon-qubit

Research

There are a wide range of papers published in the past few years that provide insight into electron-based qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2024 article published at Phys.org[3] describes a novel approach to creating qubits for quantum computers using electrons floating above liquid helium. This method, developed by researchers at the RIKEN Center for Quantum Computing and detailed in a study published in Physical Review Applied, seeks to overcome the challenges posed by defects and impurities in solid-state crystals that disrupt uniform qubit creation. These impurities create unpredictable electrical potentials, complicating the scalability of quantum computers using solid-state crystal-based qubits. To address this, the researchers propose using electrons floating in a vacuum slightly above the surface of liquid helium. This system leverages the cleanliness of the vacuum environment, which is free from the defects found in solid crystals. The electrons in this setup can be manipulated using electric fields and their spin states used for stable data storage, combining the charge state and spin state of electrons to facilitate data transfer between the two properties.

The proposed design involves one-qubit and two-qubit gates with estimations of their operational fidelity. Additionally, the setup could potentially allow the placement of over 10 million qubits in an area as small as a postage stamp through an array of tiny ferromagnetic pillars to trap the electrons above helium. The team is now looking towards experimental implementation of their theoretical proposal, aiming to achieve a significant scaling up in the number of qubits, which could revolutionize quantum computing capabilities.

A 2023 article published in physics world[4] describes a new quantum computing platform developed by researchers in South Korea that utilizes electron spins to create a three-qubit system. The team, led by Yujeong Bae, Soo-hyon Phark, and Andreas Heinrich at the Institute for Basic Science in Seoul, has successfully assembled this system atom-by-atom using a scanning tunneling microscope (STM). This quantum computing platform is notable for its capability to operate multiple spin-based quantum bits (qubits) simultaneously, an advancement that highlights the potential for spin-based qubits to be a viable option despite their current lack of advancement compared to other qubit technologies like superconducting circuits and trapped ions. The researchers utilized the precision control of an STM, which can manipulate individual atoms on a surface, to construct their qubits with atomic-scale precision.

The system is built on a magnesium oxide bilayer film and includes a “sensor” qubit made from a spin-1/2 titanium atom positioned directly under the STM tip.

[3] https://phys.org/news/2024-03-qubits-quantum-electrons-helium.html

[4] https://physicsworld.com/a/three-qubit-computing-platform-is-made-from-electron-spins/

This tip is coated with iron atoms to create a local magnetic field. Two additional "remote" qubits, also spin-1/2 titanium atoms, are placed at specific distances from the sensor qubit. These remote qubits are controlled simultaneously with the sensor through a magnetic field gradient established by nearby iron atoms. This arrangement allows the remote qubits and the sensor qubit to be controlled and manipulated individually using the STM tip through electron spin resonance techniques. The qubits are initialized by cooling to 0.4 K and applying an external magnetic field, which aligns their spins and couples them together. The entire setup has demonstrated the successful operation of single, two, and three qubit gates with good quantum coherence.

A 2022 article published in Nature[5] discusses the coherent control of electron spin qubits in silicon using a global magnetic field, a significant step towards large-scale quantum processors. Researchers employed a 3D microwave dielectric resonator to generate a uniform magnetic field across a quantum chip, enabling the simultaneous control of multiple electron spin qubits. This method represents an improvement over traditional approaches, which use local fields and face scalability challenges. Utilizing a potassium tantalate dielectric resonator, researchers achieved single spin resonance in a silicon metal-oxide-semiconductor device. This was facilitated by a global magnetic field that induced coherent Rabi oscillations, a measure of the qubit's quantum state manipulation capabilities.

The team demonstrated that the global field could coherently control qubit states, achieving Rabi frequencies over 1 MHz with a microwave input power around 20 μW. This indicates effective qubit manipulation across the entire chip using a single microwave source. Coherence times were examined through decay and echo experiments, providing insights into the qubits' operational stability and effectiveness.

A 2023 article titled "Single-electron qubits based on ring-shaped surface states on solid neon"[6] by Toshiaki Kanai, Dafei Jin, and Wei Guo, explores an innovative approach to quantum computing using single-electron charge qubits bound to solid neon surfaces. These qubits demonstrate exceptionally long coherence times, making them a promising candidate for quantum computing platforms.

The study delves into the interactions between an electron and the topography of a neon surface, particularly how bumps and valleys on the surface influence the electron's binding and quantum states. By using a theoretical framework and solving the Schrödinger equation for these surface interactions, the research reveals that electrons can be naturally trapped in ring-shaped quantum states by surface bumps, which could facilitate qubit operations. Previous experiments have shown that single-electron qubits on solid neon can achieve coherence times on the order of 0.1 ms. The electron is trapped near the neon surface by an attractive potential created by the electron's image charge induced in the neon.

[5] https://www.nature.com/articles/s41534-022-00645-w

[6] https://arxiv.org/abs/2311.02501

The study provides a comprehensive analysis of the electron's behavior when interacting with various topographical features on the neon surface. This includes the influence of bumps and valleys, which can either stabilize or destabilize the electron's quantum state. By evaluating how electrons interact with surface bumps, it is shown that these features can form toroidal trapping potentials that support stable, ring-shaped quantum states. These states are critical for potential quantum computing applications, as they allow for controlled electron spin and charge manipulations.

Conclusions

Electron spins represent one way to encode quantum information. Electrons trapped on helium as well as frozen neon offer additional methods for encoding quantum information. Electron spin states can be manipulated using magnetic and electric fields or through precisely timed microwave pulses. Silicon phosphorous also provides a substrate for creating qubits. Each of these methods has advantages and disadvantages.

Chapter 8
Fullerene-Based Quantum Computers

Introduction

Fullerenes, often referred to as "buckyballs," are spherical molecules made entirely of carbon atoms, typically arranged in a pattern resembling a soccer ball, with a typical formula of C_{60}. These molecules can encapsulate other atoms or small molecules within their structures, a property that can be exploited to create qubits for quantum computing. The fullerene itself helps block outside interference, thus improving coherence times. However, individually addressing each qubit when they are encapsulated within fullerenes is challenging, particularly in a densely packed molecular environment. Integrating fullerene-based qubits into larger, scalable quantum circuits is a major technical challenge. Creating and maintaining coherent interactions between multiple qubits across a device requires precise control and advanced fabrication techniques. As early as 2006, there were studies, including one published in the journal Nature, describing the use of fullerenes for qubits.[1]

Fullerenes

Carbon naturally occurs in the form of allotropes. As an example, both graphite and diamond are naturally occurring carbon allotropes. Fullerenes have gained wide use in nanotechnology. The C_{60} molecule was first discovered in 1985 by Robert Curl, Harold Kroto, and Richard Smally. They used lasers to recreate stellar nucleation events. The researchers accidentally discovered C_{60} closed cages in the soot from using lasers on graphite. These structures are often referred to colloquially as Buckyballs, named after Buckminster Fullerene, who was an architect who had

[1] https://www.nature.com/articles/nphys192

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_8

designed similar structures in buildings. Sir Harry Kroto was awarded the Nobel Prize in Chemistry for his methodology for engineering fullerenes. An example of a Buckyball is shown in Fig. 8.1.

The C_{60} molecule is of interest because it exhibits a high level of mechanical strength. Its strength is actually greater than that of diamond or steel. It is also the case that the C_{60} molecules will retain their original shape even after being placed under high pressure. Some tests have shown C_{60} structures surviving collisions at a speed of 15,000 miles per hour against a stainless-steel target.

The C_{60} molecule is particularly relevant to bioengineering as it can function as a transport for molecules or ions. The C_{60} molecule forms a cage that contains the molecule or ion and can transmit the content to a target. Various atoms bond well with the C^{60} cage leading to exohedral modification of the C_{60} molecule. This can be used to improve solubility of substances and is thus used in the delivery of some drugs. There are also applications wherein organic molecules such as porphyrins are attached exohedrally to the C_{60} case producing a simulation of photosynthesis.

There are other applications of C_{60} exohedral modification. For example, metal attached to C_{60} exhibits superconducting properties. However, it is the bioengineering applications that are most relevant to this current study. The aforementioned nano cages (hollow spheres) are commonly used in biomedical engineering. As one example, drug delivery can be accomplished via nano spheres. Related to C_{60} are carbon nanotubes. These structures are typically 1–20 nanometers in diameter. There are variations on nanotubes. Nanooctogans are nanotubes with octagonal ends. Nanohexagons are nanotubes with hexagonal ends. The primary three families of nanotubes are single walled, double walled, and multi-walled. These nanotubes are considered part of the family of fullerenes. A single-walled nanotube is shown in Fig. 8.2.

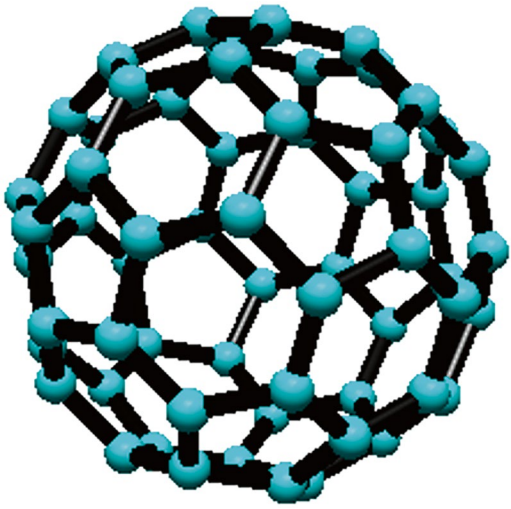


Fig. 8.1 Buckyball

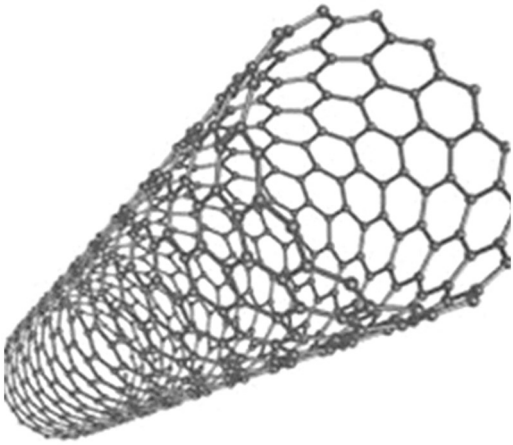


Fig. 8.2 Single-walled carbon nanotube

Various carbon structures such as nanotubes and Buckyballs have a wide range of uses. It is somewhat surprising the diversity of applications. It should be noted that research into better ways to produce fullerenes is an important area with numerous recent advances.[2] This focus is due to the numerous applications of fullerenes. Of course, the focus in this chapter is on applications to quantum computing.

Fullerene Qubits

In fullerene-based quantum computing, the qubits can be realized in several ways. One method is to use nuclear spins to represent information. Certain isotopes encapsulated within the fullerene, such as phosphorus-31, have nuclear spins that can serve as qubits. The nuclear spin states provide a basis for quantum information processing. Another method relies on electron spins to encode information. Electron spins associated with atoms trapped inside the fullerenes or with the fullerenes themselves can also be used as qubits. Electron spins are generally easier to manipulate than nuclear spins but tend to have shorter coherence times.

There are also variations of fullerenes such as fullerenes that have an additional atom, cluster, or chain enclosed within their inner voids. For instance, endohedral metallofullerenes (for example, $N@C_{60}$, where a nitrogen atom is trapped inside a C_{60} molecule) can exhibit unique electronic properties that are useful for quantum computing. These encapsulated atoms can interact with the fullerene's electron cloud, leading to specific electronic structures ideal for quantum manipulation.

The relatively isolated environment inside the fullerene sphere helps to shield the encapsulated atoms from external magnetic noise, potentially leading to longer

[2] https://www.proactiveinvestors.com/companies/news/1018141/large-scale-fullerene-production-opens-up-world-of-possibilities-in-medicine-electronics-1018141.html

coherence times. This isolation is critical in maintaining the quantum state integrity necessary for quantum computation. Manipulating the qubits in fullerene-based quantum computers can be achieved through various methods. One method is radio frequency pulses which can be used to manipulate nuclear or electron spins within the fullerene. Electromagnetic fields can also be used to control the electron spins of the encapsulated atoms or the fullerene itself.

In some configurations, laser light can be used to read out or manipulate the quantum states. Lasers can be used to optically pump the system, driving the encapsulated atom or molecule into a specific spin state. This is achieved by tuning the laser to a specific wavelength that excites only particular transitions within the fullerene, effectively preparing the qubit in a desired initial state. Through carefully timed and controlled laser pulses, the quantum state of the encapsulated atomic or molecular spins can be coherently manipulated. This manipulation often involves inducing Rabi oscillations, where the state of the qubit oscillates between the basis states under the influence of an oscillating electromagnetic field provided by the laser. For operations involving more than one qubit, lasers can mediate effective interactions between separate fullerene molecules. For example, by using laser-induced dipole-dipole interactions or by entangling the spins via photon-mediated interactions, where the emission from one qubit can affect the state of another.

Rabi oscillations are a fundamental phenomenon in quantum mechanics, describing the coherent oscillation of a two-level quantum system (such as an atom, molecule, or qubit) when subjected to an external oscillatory driving field, typically an electromagnetic field like a laser or microwave. They are relevant to various physical qubit implementations and thus will be discussed in different chapters of this book. A two-level system consists of two quantum states, usually referred to as the ground state $|0\rangle$ and the excited state $|1\rangle$. The system can transition between these two states when exposed to a resonant driving field. The external field (e.g., a laser or microwave) has a frequency that matches the energy difference between the two states, inducing transitions between them. The field's strength and frequency are critical in determining the dynamics of the oscillations. The Rabi frequency is a measure of the strength of the interaction between the driving field and the two-level system. The Rabi frequency determines the rate at which the system oscillates between the ground and excited states.

Light can also be used in reading the qubit. One method is detecting the fluorescence emitted by the system when illuminated by a laser. The spin state of the encapsulated atom affects its optical properties, such as the frequency or intensity of the fluorescence, allowing the spin state to be inferred from the optical measurement. Lasers can also be used to probe resonance conditions in the fullerene. Changes in the resonance frequency, induced by the spin state of the qubit, can be detected by monitoring the absorption or transmission of laser light at various frequencies. In systems where multiple fullerene qubits are addressed by lasers, isolating the laser interaction to a single qubit without affecting neighboring qubits is challenging. This requires sophisticated focusing techniques and potentially the integration of optical elements at the nanoscale.

Synthesizing endohedral fullerenes with high purity and in sufficient quantities is technically challenging and costly. Additionally, isolating individual fullerene molecules for use in quantum computing requires advanced techniques. Integrating fullerene-based qubits into larger quantum circuits involves bridging molecular quantum computing with conventional or emerging quantum technologies. This might involve hybrid systems where molecular qubits interact with photonic or superconducting elements. Building a scalable quantum computing system using fullerenes requires not only precise control over individual qubits but also a reliable means of interconnecting qubits and transmitting quantum information between them.

Quantum Dot Qubits

While this topic is not, strictly speaking, about fullerene qubits, it is very closely related. A device that is only of nanometer scale in 1 dimension is termed a Quantum Well. A device that is of nanometer scale in 2 dimensions is termed a Quantum Wire. A device that is of nanometer scale in all 3 dimensions is termed a Quantum Dot. The scale of the device in each of the three dimensions is what defines the appropriate nomenclature. Because of their small size, quantum dots have quantum mechanical properties, which makes them suitable for various applications including quantum computing.

Quantum dots have discrete, tunable energy levels. By changing the size of the quantum dot or applying external electric or magnetic fields, one can control the energy levels. This adjustability allows for precise control over the quantum states of the dots, which is crucial for their function as qubits. Unlike atoms that require laser fields for manipulation, quantum dots can be controlled using electric fields, which can be easier to implement with current semiconductor technology. This makes quantum dots well-suited for integration into existing electronic architectures. Quantum dots can potentially be arranged in arrays and scaled up using fabrication techniques common in the semiconductor industry, promising for the development of more complex quantum computing systems.

The quantum information is encoded in the charge configuration of the electrons in the quantum dots. For example, the presence or absence of an electron in a specific quantum dot can represent the binary states 0 and 1. Quantum dots can be tuned to allow for tunneling of electrons between them, facilitating the initialization, manipulation, and measurement of quantum states. It is also possible to encode the quantum information in the spin state (up or down) of an electron within a quantum dot. Spin qubits benefit from longer coherence times compared to charge qubits because they are less sensitive to electrical noise. Control over spin states can be achieved using magnetic fields or spin-orbit interactions, and single-qubit gates and two-qubit gates can be implemented with high precision. The state of a quantum dot qubit can be read using techniques like charge sensing and spin-to-charge conversion. Charge sensing involves measuring the electrical environment around the quantum dot to infer the presence of electrons, whereas spin-to-charge conversion

leverages changes in charge configuration that depend on the spin state, detected via a nearby quantum point contact or single-electron transistor.

Quantum dots are subject to various types of decoherence, such as dephasing from charge noise and spin relaxation through interactions with nuclear spins in the host material. Advanced material engineering and isotopic purification are strategies used to mitigate these effects. The properties of quantum dots can vary from one dot to another due to differences in size, shape, and material defects, which poses a challenge for uniformity in quantum dot arrays. As systems scale up, the complexity of integrating and individually addressing multiple quantum dots becomes significant. Developing multiplexing strategies and improving fabrication techniques are active areas of research to overcome these challenges.

A preprint from 2024 describes a study using spin qubits in semiconductor quantum dots.[3] The paper outlines how quantum dots, created within semiconductors, can trap electrons or holes in a potential well, giving them quantized energy levels. These quantized levels allow for the creation of spin qubits, where quantum information is encoded in the spin states of these confined charges. Initial experiments predominantly used gallium arsenide (GaAs) materials, but there has been a significant shift towards using silicon-based materials like silicon-metal-oxide-semiconductor (Si-MOS) and silicon/silicon-germanium (Si/SiGe) heterostructures due to their improved properties for qubit development. Spin qubits are controlled by magnetic and electric fields, which can be adjusted to manipulate the qubit states. However, these qubits are subject to decoherence from environmental noise, impacting their quantum state over time. The article discusses how different types of spin qubits have different mechanisms for control and vary in their susceptibility to noise.

In addition to spin-based quantum dots there are spatial-based quantum dot qubits. Spatial-based quantum dot qubits utilize the discrete electronic states of quantum dots to encode and manipulate quantum information. Quantum dots are semiconductor nanostructures that confine electrons or holes in all three spatial dimensions, leading to quantized energy levels similar to those found in atoms. This confinement and the ability to precisely control the quantum states make quantum dots promising candidates for qubits in quantum computing.

Quantum states can be represented several different ways in quantum dots. The presence or absence of an electron in a quantum dot can be used to represent qubit states. This is particularly useful in double quantum dot systems where the charge state can be controlled by external voltages. Charge qubits are manipulated using electric fields, which control the tunneling of electrons between quantum dots. Excitons are bound states of an electron and a hole within a quantum dot. The presence or absence of an exciton can represent qubit states. Exciton qubits are typically controlled using optical fields to create and annihilate excitons.

[3] https://arxiv.org/abs/2204.04261

Metallic-Like Carbon Nanospheres

Metallic-like carbon nanospheres (MCNs) represent an innovative approach in the realm of quantum computing, leveraging the unique properties of carbon nanostructures. While this concept is still largely theoretical and at the forefront of research, it offers intriguing possibilities for creating qubits and developing quantum computing technologies. Carbon nanospheres are spherical nanostructures made entirely of carbon atoms. When these nanospheres exhibit metallic-like properties, they have high electrical conductivity and unique electronic characteristics. These nanospheres also exhibit high thermal stability, mechanical strength, and excellent electrical conductivity, which are essential for quantum computing applications.

The spin states of electrons confined within the carbon nanospheres can be used as qubits. These spin states can be manipulated using magnetic fields and detected through various spectroscopic techniques. The nuclear spin of atoms within or near the carbon nanospheres can also serve as qubits, providing another degree of freedom for quantum information processing. Operations on individual qubits can be performed using localized magnetic fields or microwave pulses to manipulate the spin states. Entangling gates can be implemented by exploiting the interactions between neighboring carbon nanospheres or by using external fields to mediate interactions. Techniques such as electron spin resonance (ESR) and nuclear magnetic resonance (NMR) are commonly used for readout of values (i.e., measurement).

Research

There are numerous research papers published in the past few years that provide insight into fullerene-based qubits. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

As early as 2018, there was published research in the Journal Chemical Science discussing qubits created out of a Sc^3C^2@C80 fullerene.[4] The article focused on Sc3C2@C80 which is composed of a metallic cluster inside a carbon cage. This configuration isolates the core from environmental disturbances, potentially enhancing its quantum coherence properties. The study shows how Sc3C2@C80 in CS2 solution exhibits quantum coherence from 5 K to room temperature. At temperatures below about 140 K, the molecule shows a single broad electron paramagnetic resonance (EPR) line indicating limited core movement. Above 140 K, the core becomes mobile, displaying 22 distinct, homogeneously broadened EPR lines due to isotropic hyperfine coupling. The phase memory time (a measure of quantum coherence) reaches a maximum of 17.2 ms at 10 K. At 200 K, this time drops

[4] https://www.google.com/

significantly to around 139 ns, still sufficient for performing quantum state manipulations. At lower temperatures (below 140 K), the spin-lattice relaxation and phase-memory time are longer, indicating stable quantum states suitable for certain quantum computing operations.

A paper published in 2021 described manipulating quantum phase interference in a fullerene-based qutrit.[5] This study reported on the photoexcited triplet state of a fullerene molecule (C70) as a three-level quantum system (qutrit) for quantum information processing (QIP). This study advances the ability to control and manipulate high-spin molecular systems which are seen as promising candidates for quantum computing due to their multilevel nature allowing for more complex quantum states and computations. The researchers utilized a C70 fullerene molecule, excited into a triplet state via photoexcitation, as a qutrit. They achieved coherent manipulation using pulsed electron paramagnetic resonance, demonstrating the system's stability and potential for quantum operations. Significant results include the observation of quantum phase interference, which demonstrates the coherent superposition of quantum states within the qutrit. This interference is key for quantum computing operations, as it can be used to perform complex calculations.

A 2023 article discussed the enhancement of quantum coherence times in molecular electron spin qubits.[6] The study demonstrates the use of mechanochemical methods to integrate copper (II)-porphyrins at a 10% dilution into zirconium-based Metal-Organic Frameworks (MOFs), specifically targeting the PCN-223 and MOF-525 structures. The introduction of fullerene (C_{60}) during the synthesis promotes the formation of the PCN-223 framework while improving electron spin-lattice and phase-memory relaxation times (T1 and Tm), attributed to the reduction of nuclear spin-bearing solvent guests and modulation of the phonon density of states. The incorporation of fullerene not only templated the MOF structure but also significantly enhanced the relaxation times, suggesting better performance of the qubits. This is due to fullerene's role in stiffening the framework and reducing magnetic noise, which are essential for maintaining coherence in quantum computing applications.

The study employed techniques such as continuous wave and pulse electron spin resonance (ESR) spectroscopy to analyze the quantum states and interactions within the MOF structures. The results indicated that fullerene inclusion leads to noticeable improvements in the operational characteristics of the qubits. The research indicates that Metal-Organic Frameworks can be successfully used to enhance the performance of molecular spin qubits by controlling the internal environment through the strategic insertion of fullerene. This approach offers a potential pathway to more robust and reliable quantum computing elements.

[5] https://www.nature.com/articles/s41534-021-00362-w

[6] https://pubs.rsc.org/en/content/articlepdf/2023/sc/d3sc03089j

Conclusions

Fullerene-based quantum computers are a part of a broader category of molecular and solid-state quantum computing platforms that offer unique advantages, especially in terms of potentially long coherence times and the natural quantization of nuclear and electron spins. The field is still nascent, with ongoing research needed to overcome substantial practical challenges related to qubit manipulation, scalability, and integration into functional quantum computing architectures.

Chapter 9
Quantum Annealing

Introduction

At the heart of D-Waves approach is the superconducting qubit (SQUID). This is the equivalent of the classical computer's transistor. The term SQUID comes from the fuller name, Superconducting QUantum Interference Device. SQUIDs use the Josephson junction. There are two types of SQUID, the direct current (DC) and radio frequency (RF). A Josephson junction is something that occurs when two superconductors are in close proximity. This produces a current, in fact a supercurrent. This was discussed in detail in *Chap. 2: Superconducting Quantum Computers*. While superconducting and Josephson junctions are associated with superconducting qubits, the D-Wave SQUID approach uses quantum annealing rather than quantum gates.

D-Wave's quantum annealers are particularly suited for specific types of problems like optimization, machine learning, material science, and logistics. They offer a quantum computing-as-a-service model, allowing users to access their quantum processors through the cloud. There has been some debate in the scientific community about the extent to which D-Wave's machines exploit quantum mechanics to offer a computational advantage over classical computers. Some experiments suggest a quantum speedup, while others indicate that for certain tasks, traditional algorithms on classical computers might be more effective.

The D-Wave Approach

The approach used by D-Wave is different than the quantum gates that are used by most quantum computing vendors. Some would argue that this is not "true" quantum computing. However, it does allow practical implementations of quantum

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_9

computing now. It is also the case that since this approach does not depend on quantum gates, larger numbers of qubits are usually obtained. In this section, the specific aspects of this approach to quantum computing will be explored.

Adiabatic Quantum Computing

Adiabatic quantum computing (AQC) is a unique approach to quantum computing that leverages the quantum mechanical phenomenon of adiabatic evolution. This model of computation is distinct from the more commonly known gate-based quantum computing. Adiabatic quantum computing is based on *Adiabatic Theorem* of quantum mechanics which states that a quantum system in its ground state will remain in its ground state provided that the Hamiltonian H is varied slowly enough. Also, a quantum system whose energies are quantized that starts in the nth energy state will exist in the nth energy state provided that the Hamiltonian is varied slowly enough. Vary the Hamiltonian slowly from an initial to final state so that it acts as though a unitary transformation occurred on the initial state, bringing it to a final state during some time T.

For those readers who may be unfamiliar with the Hamiltonian, a brief description is provided here. In physics, the Hamiltonian is a fundamental concept used to describe the total energy of a system. It plays a central role in both classical mechanics and quantum mechanics. The Hamiltonian is typically denoted by H and is used to formulate the equations of motion for a system. The Hamiltonian represents the total energy of a system, which is the sum of its kinetic energy (T) and potential energy (V). quantum mechanics, the Hamiltonian is an operator corresponding to the total energy of the system. The Hamiltonian operator $\hat{H}$ acts on the wave function ψ of the system. The time-independent Schrödinger equation is:

$$\hat{H}\psi = E\psi$$

In that equation, E is the energy eigenvalue corresponding to the eigenstate ψ. For the reader who may need a review of eigenvalues and eigenstates, please consult appendix B. The Hamiltonian is a central concept in both classical and quantum mechanics, representing the total energy of a system. In classical mechanics, it provides a framework for formulating the equations of motion. In quantum mechanics, the Hamiltonian operator determines the time evolution and energy states of quantum systems. In quantum computing, Hamiltonians are used to encode problems and design quantum gates, playing a crucial role in the functioning of quantum algorithms and quantum annealers.

In adiabatic quantum computing, the problem to be solved is encoded into the ground state of an initial Hamiltonian, which is relatively simple and easy to prepare. The Hamiltonian is then adiabatically transformed into a final Hamiltonian, whose ground state encodes the solution to the problem. The evolution between

these Hamiltonians is controlled and slow, ensuring the system remains in its ground state. Adiabatic quantum computing is closely related to, and often confused with, quantum annealing. Quantum annealing is a specific type of adiabatic quantum computing used mainly for solving optimization problems and is implemented in hardware like the D-Wave systems. It specifically uses thermal fluctuations to aid in finding the lowest energy state of the Hamiltonian, which represents the optimal solution.

Quantum Annealing

Quantum annealing is a specialized quantum computing technique used to solve combinatorial optimization problems. It is particularly well-suited for certain types of optimization tasks, such as finding the minimum or maximum value of a mathematical function with many variables. The process involves mapping the problem to be solved onto a set of quantum bits, or qubits, and then applying quantum annealing algorithms to find the optimal solution. The idea behind quantum annealing is inspired by the physical concept of annealing in metallurgy, where a metal is heated and then slowly cooled to reduce its defects and reach a more ordered state. The term quantum annealing was first proposed by Apolloni, Bianch, and De Falco in 1988.[1]

In quantum annealing, the system starts in a state where all qubits are in a superposition of states, meaning they represent all possible solutions to the problem simultaneously. The system evolves using according to the time-dependent Schrödinger equation shown here:

$$i\hbar \frac{d}{dt}\left|\psi(t)\right> = \hat{H}\left|\psi(t)\right>$$

In the preceding equation, which no doubt many readers are already quite familiar with, the t represents time; $|\psi>$ is the state vector of the system in question; $\hat{H}$ is the Hamiltonian of the system. As the system evolves the amplitudes of the candidate states change.

As the system evolves, it gradually settles into a state that corresponds to the optimal solution of the problem being solved. This settling process is akin to the gradual cooling of a metal during annealing. One of the most well-known implementations of quantum annealing is provided by D-Wave Systems, which builds quantum annealing machines known as D-Wave quantum computers. These machines have been used to tackle various optimization problems in fields such as finance, logistics, and machine learning. It is important to note that quantum

[1] https://www.researchgate.net/publication/250309136_A_numerical_implementation_of_quantum_annealing

annealing is not a universal quantum computing technique and is best suited for specific types of optimization problems. Other quantum computing approaches, such as gate-based quantum computing, may be better suited for different types of problems. It may be interesting to note that annealing is a term from metallurgy, annealing is a heat treatment that alters the microstructure of a material causing changes in properties such as strength and hardness. Commonly done by heating the material until it glows then letting it slowly cool to room temperature.

One of the main advantages of adiabatic quantum computing is that it can be robust against certain types of operational errors and decoherence (loss of quantum information due to interaction with the environment), as the system remains in the lowest energy state throughout the process. Implementing adiabatic quantum computing requires systems that can precisely control the evolution of Hamiltonians and maintain coherence throughout the adiabatic transition. Current implementations use superconducting circuits and involve sophisticated control over quantum states. While adiabatic quantum computing promises robustness and efficiency for certain types of problems, its physical realization is challenging. The requirement for slow evolution to maintain adiabatic conditions can lead to long computation times compared to gate-based quantum computing. Moreover, the design of suitable Hamiltonians for specific problems is a non-trivial task that requires deep understanding of both quantum mechanics and the problem at hand.

D-Waves Products

Due to the reliance on adiabatic quantum computing rather than quantum gates, D-Wave has been able to achieve more qubits than other manufacturers. As early as August of 2015, they announced their D-Wave 2X computer with 1000 qubits. The D-Wave 2DX utilized D-Waves Chimera graph architecture. The Chimera graph is built from a basic unit called a cell. Each cell contains a set of qubits which are arranged in a bipartite graph. Each qubit in one set is connected to all qubits in the other set within the same cell. This is akin to a complete bipartite graph $K_{4,4}$, where each side of the partition has four qubits, resulting in a total of eight qubits per cell in the original D-Wave designs. Cells are arranged in a grid. Qubits in the same row or column are connected across adjacent cells, allowing interactions between qubits in different cells but aligned either vertically or horizontally.

The Chimera graph's structure dictates how these problems can be mapped onto the qubits and their interactions. The arrangement allows the quantum annealer to effectively find low-energy configurations corresponding to optimal or near-optimal solutions to the problem at hand. The nature of the connections—both within a cell and between cells—facilitates this process by supporting complex interaction patterns necessary for solving intricate optimization problems. Although versatile for many optimization problems, the Chimera graph is relatively sparse compared to other potential quantum architectures, meaning it does not allow for all-to-all connectivity among qubits. This sparsity can limit the types of problems that can be

directly mapped onto the system without extensive reconfiguration or embedding strategies.

This requires some exploration of what a Chimera graph is. A Chimera graph is a specific type of graph structure used primarily in the context of quantum computing. A Chimera graph represents a hybrid network topology that combines elements of two different types of graphs to facilitate efficient quantum computation. The Chimera graph is based on a two-dimensional lattice structure where each node (also called a qubit in the context of quantum computing) is connected to a set of neighboring nodes. This forms a lattice structure. The graph consists of multiple unit cells, each typically containing a small number of qubits. A common configuration is a 4x4 unit cell, meaning each unit cell has 4 rows and 4 columns of qubits. Within each unit cell, qubits are interconnected in a bipartite fashion, meaning qubits in one set are only connected to qubits in the other set. Unit cells are also connected to their neighboring cells, creating a larger, more complex graph. This allows for the embedding of more complicated problems into the quantum annealer.

A Chimera graph C (m, n, L) can be described using three parameters. The first is the number of unit cells in the vertical direction denoted by m. The second parameter is the number of unit cells in the horizontal direction, denoted by n. The third parameter is the number of qubits per side in a unit cell, denoted by L. For example, in a Chimera graph C (3,3,4), there are 3×3 unit cells, each containing 4 qubits per side, resulting in a total of $3 \times 3 \times 4 \times 4 = 144$ qubits.

Since 2019, D-Waves flagship product has been their Advantage system, currently in its 5th generation. The 5th generation of Advantages boasts over 5000 qubits.[2] These systems utilize the Pegasus graph topology, rather than the Chimera graph that D-Wave previously used. Like the Chimera graph, the Pegasus graph is structured around a grid of cells, but with a crucial difference in the arrangement and connectivity of the qubits. Each cell in Pegasus contains more qubits than in Chimera, typically arranged to increase inter-qubit connections. In Pegasus, qubits are arranged in a way that each qubit connects not just to immediate neighbors within a grid but also to additional qubits in a pattern that allows more densely packed connectivity. This results in each qubit having 15 connections, compared to 6 in the Chimera architecture.

The Pegasus graph is a specific type of graph topology that represents an evolution from the earlier Chimera graph. Each qubit in the Pegasus graph is connected to 15 other qubits, compared to 6 connections per qubit in the Chimera graph. This increased connectivity allows for more complex and larger problem embeddings with fewer intermediate qubits. The Pegasus graph consists of unit cells arranged in a grid. Each unit cell contains qubits arranged in a pattern that allows for both intra-cell and inter-cell connections. The connections between qubits in different unit cells follow a specific pattern that enhances the overall connectivity of the graph. The qubits are arranged in a manner that maximizes the potential for embedding

[2] https://www.dwavesys.com/solutions-and-products/systems/

complex problems directly onto the hardware, reducing the need for additional qubits to represent logical connections between qubits in the original problem.

The Pegasus graph is designed to maximize the use of available qubits and couplers in the quantum processing unit (QPU). It extends the basic Chimera cell by enhancing vertical and horizontal connections and introducing new diagonal connections that cross over multiple traditional Chimera cells. The increased connectivity of the Pegasus topology enables a more direct and efficient mapping of complex optimization problems. This is because the additional connections reduce the need for intermediary qubits and couplings, which in turn can simplify the problem embedding process and potentially yield better performance. The greater number of connections per qubit allows the Pegasus architecture to handle a broader array of problem types, including those requiring more complex interaction patterns between variables.

As you can undoubtedly imagine, the Pegasus architecture, like all architectures, offers both advantages and disadvantages. The dense connectivity reduces the overhead required to represent problem constraints and interactions, allowing for a more efficient use of the quantum hardware. With more connections per qubit, the Pegasus architecture can scale more effectively to larger problem sizes compared to the Chimera graph. With increased connectivity comes the challenge of managing and controlling a denser network of qubits. Additionally, error correction becomes more challenging as the number of interactions increases. Figure 9.1 depicts a chart from D-Wave's white paper on the Advantage product,[3] compares Chimera and Pegasus architecture.

Figure 9.2 is a picture of the D-Wave Advantage computers.

The D-Wave Advantage system represents a significant leap forward in quantum annealing technology, offering a high qubit count, improved connectivity through the Pegasus topology, and enhanced performance. It is designed to solve complex optimization and sampling problems across various industries, leveraging a hybrid quantum-classical approach for improved efficiency. The Pegasus topology and increased connectivity simplify problem embedding, enhancing performance and solution quality.

Fig. 9.1 D-Wave architecture comparison

	2000Q	Advantage
Graph topology	Chimera	Pegasus
Graph size	C16	P16
Number of qubits	> 2000	> 5000
Number of couplers	> 6000	$> 35,000$
Couplers per qubit	6	15

[3] https://www.dwavesys.com/resources/white-paper/the-d-wave-advantage-system-an-overview/

Fig. 9.2 D-Wave advantage

Other Quantum Annealing Endeavors

While D-Wave is the most prominent company involved in quantum annealing, there are others. In this section, these other quantum annealing endeavors will be briefly examined.

Qilimanjaro Tech

One such company is Qilimanjaro Tech.[4] This company does both quantum annealing and gate based superconducting quantum computing. They describe their annealing process as "Rather than slicing problems into a set of discrete operations, as in gate-based processors, which may introduce several control errors, analog quantum computers encode the problem in the quantum description (e.g., Hamiltonian) of the device and let the system evolve with time under continuous control. Importantly, they are a promising alternative to bypass the need for heavy error correction protocols and their associated qubit overheads."

The Qilimanjaro system leverages a coherent quantum annealing approach, which is designed to address complex optimization problems and enable quantum simulations that are classically intractable. Qilimanjaro's quantum annealer utilizes superconducting qubits with long coherence times, up to 1000 times longer than other implementations. The architecture of the annealer ensures high connectivity

[4] https://www.qilimanjaro.tech/

between qubits, which enhances its capability to solve complex optimization problems by facilitating more efficient quantum qubit-qubit interactions. The Qilmanjaro quantum annealer is part of a larger initiative to integrate quantum computing with High Performance Computing systems, such as the MareNostrum 5 supercomputer.

Fujitsu

Fujitsu offers its Digital Annealer.[5] They have focused on a range of applications including route optimization, logistics, and industrial optimization. Unlike quantum annealers, the Digital Annealer uses classical digital circuits to simulate the quantum annealing process, offering high-speed performance and precision for optimization tasks without the need for quantum hardware. The Digital Annealer mimics the principles of quantum annealing using classical hardware. It solves optimization problems by simulating the behavior of quantum systems, particularly the process of finding the ground state of a system. Leveraging advanced digital circuits, the Digital Annealer can perform computations at high speeds, significantly faster than traditional computing methods for certain types of optimization problems. They also provide an excellent description of quantum annealing:[6]

> When the algorithm has a target function (for example, energy) and a range of possible outcomes, annealing is looking for an optimal state, which in this example would be minimal energy. Simulated annealing finds this by a random 'walk' through the possible outcomes while simultaneously cooling down the system f we slow this right down for the purposes of explanation, what happens is that the simulated annealing algorithm starts with a high energy and temperature and randomly takes a step in some direction to one of the possible outcomes. If the energy measured at this second step is lower than at the starting step, then it is accepted. The system temperature is now a little lower and the algorithm takes the next random step.
>
> If that step leads to a higher energy measurement, then it could also be accepted. The decision is made by a random experiment, where the acceptance probability decreases with higher energy measurements and lower temperatures. Within this process, some steps will end with a new minimum energy. These may be 'local minima'—the lowest found in that round of measurements, or they might be the lowest result possible from the entire field of possibilities, which is known as the 'global minimum'. However, the current existing architectures often get trapped in the local minimum and leads to more time required to obtain a sub-optimal solution.

[5] https://www.fujitsu.com/global/services/business-services/digital-annealer/

[6] https://sp.ts.fujitsu.com/dmsp/Publications/public/Digital_Annealer_White_Book.pdf

Hitachi

Hitachi is another company utilizing quantum annealing. While it leverages concepts inspired by quantum annealing, it uses classical computing components, particularly CMOS (Complementary Metal-Oxide-Semiconductor) technology, to perform its computations. This allows Hitachi to harness some of the benefits associated with quantum annealing without the need for a full quantum computer infrastructure. The CMOS Annealing Machine mimics the behavior of quantum annealing using classical hardware. It applies analog circuitry to perform energy minimization processes similar to those used in quantum annealers. The machine explores the energy landscape of an optimization problem to find its global minimum, effectively solving combinatorial optimization problems. While the CMOS Annealing Machine is inspired by quantum annealing, it does not provide the same potential advantages as true quantum annealers, such as exploiting quantum superposition and entanglement. The machine is particularly suited for combinatorial optimization problems, which means it may not be as versatile for other types of computational tasks compared to general-purpose quantum computers.

NEC

NEC is another company working on quantum annealing. NEC Corporation has been actively developing quantum annealing technology, aiming to solve complex combinatorial optimization problems efficiently. NEC utilizes superconducting parametron technology to create qubits with long coherence times, which is crucial for maintaining the quantum state needed for accurate and high-speed quantum annealing computations. This technology enables NEC to develop quantum annealing machines capable of solving complex optimization problems by maintaining a stable quantum state over extended periods.

Superconducting parametron technology is a type of quantum computing architecture that leverages superconducting circuits to create and manipulate qubits. This technology is based on the principles of superconductivity and parametric oscillation, and it aims to achieve high coherence times and robust qubit connectivity, making it suitable for applications in quantum annealing and other forms of quantum computing. Parametric oscillation involves varying a parameter of a system (such as the inductance or capacitance in a circuit) to induce oscillations. In the context of superconducting parametron technology, this is used to control qubit states. By modulating the circuit at twice the natural resonant frequency, it is possible to create two stable oscillation states (representing the qubit states 0 and 1). Superconducting parametrons use the oscillation states of a superconducting circuit to represent qubits. These qubits can be manipulated using microwave pulses.

A typical superconducting parametron qubit consists of a superconducting coil and a capacitor forming a resonant circuit. The circuit operates at cryogenic temperatures to maintain superconductivity. The architecture allows for high connectivity between qubits, facilitating complex quantum operations and the implementation of all-to-all connected networks essential for quantum annealing. NEC's quantum annealers are designed to address complex combinatorial optimization problems in various fields, including logistics, finance, and disaster management.

NEC has developed a unit cell structure that allows for scaling up to a fully connected quantum annealing architecture. This structure ensures that the system can handle large-scale problems by maintaining logical connectivity between qubits. While awaiting the realization of large-scale quantum annealing machines, NEC has introduced a quantum-inspired simulated annealing service that leverages their vector supercomputer, SX-Aurora TSUBASA. This service uses advanced algorithms to solve combinatorial optimization problems efficiently by narrowing down the search space for potential solutions. NEC collaborates with various institutions such as Tohoku University and the National Institute of Advanced Industrial Science and Technology (AIST). Their joint research includes developing high-performance computing platforms that integrate both quantum and simulated quantum annealing capabilities. This initiative aims to enhance the performance of vector supercomputers and explore the unique advantages of quantum annealing for solving real-world problems like optimal evacuation routes during natural disasters.

Research

There are numerous research papers published in the past few years that provide insight into quantum annealing and adiabatic quantum computing. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2022 article published in Nature describes parallel quantum annealing.[7] The article describes a novel approach to utilizing quantum annealers for solving NP-hard problems more efficiently by executing multiple problem instances in parallel on a single quantum annealer chip. The articles describe a method that allows multiple independent problems, either similar or different, to be solved simultaneously during a single annealing cycle. This approach utilizes the available qubits more effectively by embedding multiple problem instances onto the quantum processing unit (QPU), thereby reducing the Time-To-Solution (TTS) metric significantly compared to sequential problem solving.

[7] https://www.nature.com/articles/s41598-022-08394-8

Experiments were conducted using the D-Wave quantum annealer to solve instances of the Maximum Clique problem. The results demonstrated that while there might be a slight decrease in the solution quality of individual problems when solved in parallel, the overall speedup in TTS is substantial, making this method advantageous for scenarios requiring solutions to multiple problem instances quickly. The process involves mapping problem graphs onto the qubit connectivity graph of the annealer, which often requires a minor embedding due to the mismatch between the problem and hardware graph structures. The method leverages the concept that independent quadratic binary optimization problems (QUBOs) can be solved in parallel without interfering with each other's solutions.

A 2021 article published by the American Physical Society[8] discusses the application of quantum annealing controls to optimize portfolio management using Markowitz portfolio theory. The article begins with a general discussion of quantum annealing (QA) as a method for finding optimal solutions to computational problems, emphasizing its use in unconstrained optimization via the encoding of problem cost functions in a Hamiltonian. The focus of the study is on the performance of various quantum annealing controls when applied to portfolio optimization. This includes experiments on both forward and reverse annealing methods using the D-Wave 2000Q quantum annealer.

Markowitz portfolio theory, also known as the Modern Portfolio Theory (MPT), was introduced by Harry Markowitz in his 1952 paper "Portfolio Selection." This foundational theory of finance revolutionized the way investors think about portfolios, risk, and return. Markowitz's theory posits that an investor's decision should not only focus on maximizing returns but also on minimizing risk, which is quantified as the variance (or standard deviation) of portfolio returns. One of the central tenets of MPT is that diversification can reduce the risk of a portfolio. By holding a mix of assets that do not perfectly correlate with each other, an investor can reduce the overall volatility of the portfolio. This concept involves creating a set of optimal portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return. These portfolios lie on a curve called the efficient frontier in the risk-return space. The theory typically involves solving an optimization problem where the objective is to minimize the portfolio's variance subject to a target expected return.

The authors use Markowitz portfolio theory to form a class of unconstrained optimization problems. They test different preprocessing and annealing controls available on the D-Wave 2000Q to evaluate their impact on computational accuracy and the probability of finding optimal solutions. The research compares empirical results from the quantum annealer against computational ground truths for several portfolio optimization instances. It investigates the effect of control variations such as embedding techniques, annealing schedules, and other QA parameters on the probability of success and chain breaks.

[8] https://journals.aps.org/prapplied/abstract/10.1103/PhysRevApplied.15.014012

Conclusions

Adiabatic quantum computing works by encoding problems to be solved into the ground state of an initial Hamiltonian. The Hamiltonian is then adiabatically transformed into a final Hamiltonian, whose ground state encodes the solution to the problem. Adiabatic quantum computing is closely related to quantum annealing. Quantum annealing is a specific type of adiabatic quantum computing used mainly for solving optimization problems. This process does not use quantum gates but has had very promising results with solving practical problems.

Chapter 10
Topological Quantum Computing

Introduction

Topological quantum computing was proposed in 1997 by physicist Alexei Kitaev. Topological quantum computers are essentially equivalent in computational power to other models of quantum computation. Especially topological quantum computing is essentially equivalent to the quantum circuit model and to the quantum Turing machine model Topological quantum computing (TQC) is a promising approach to quantum computing that relies on the unique properties of certain exotic states of matter known as topological states. Unlike traditional quantum computing paradigms, which use individual quantum bits (qubits) to encode and process information, topological quantum computing employs so-called non-Abelian anyons, which emerge as quasiparticles in certain two-dimensional systems such as certain fractional quantum Hall states or topological superconductors.

The Quantum Hall Effect (QHE) is a quantum phenomenon observed in two-dimensional electron systems that are subjected to low temperatures and strong magnetic fields. This effect was discovered by Klaus von Klitzing in 1980, for which he was awarded the Nobel Prize in Physics in 1985. The QHE is not only significant for its intriguing physical properties but also for its practical applications, including the establishment of a high-precision standard for electrical resistance. The Hall effect will be described in more detail later in this chapter.

Topological order is a special type of order in quantum systems that arises from long-range entanglement and cannot be characterized by local order parameters. It is robust against local perturbations and can support exotic quasiparticles called anyons. Anyons are exotic quasiparticles that emerge in two-dimensional systems with topological order. They possess fractional statistics, meaning that when two anyons are exchanged, the wave function of the system acquires a phase that depends on the order of exchange. Anyons can be either Abelian or non-Abelian. Non-Abelian anyons, which are of particular interest in topological quantum computing,

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_10

have the property that the result of exchanging them depends not only on the order of exchange but also on the specific paths taken by the anyons.

In topological quantum computing, information is encoded in the non-Abelian anyonic states, and computation is performed by manipulating these anyons through a process called braiding. Braiding involves moving the anyons around each other in a carefully controlled manner to perform quantum gates. Since the outcome of braiding non-Abelian anyons depends on their paths and the order of exchange, it enables fault-tolerant quantum computation. By braiding non-Abelian anyons, it is possible to perform universal quantum computation. Certain sequences of braid operations correspond to quantum gates, such as the NOT gate or the Controlled-NOT gate, which are essential building blocks for quantum algorithms. One of the key advantages of topological quantum computing is its potential for fault tolerance. Due to the topological protection provided by non-Abelian anyons, errors caused by local perturbations or noise can be suppressed, making it possible to build robust quantum computers.

Topological quantum computing is still a highly theoretical field, and experimental realizations of non-Abelian anyons and topological quantum computation are ongoing areas of research. However, the unique properties of topological states of matter offer promising avenues for overcoming some of the challenges faced by traditional quantum computing approaches.

While topological quantum computing is primarily research rather than practical implementations, in March 2022, Microsoft made advances towards topological qubits.[1,2] One of the most appealing aspects of quantum braiding and topological quantum computing is their inherent fault tolerance. Since the information is stored in the global topological properties of the system (the overall pattern of the braiding), it is significantly less sensitive to local errors and disturbances, such as those caused by decoherence or operational imperfections.

Topological quantum computing provides inherent protection against errors through the topological nature of anyons and braiding. This reduces the need for complex error correction codes. The information is stored globally, which makes the system less susceptible to local noise and decoherence. Topological qubits can potentially be scaled more easily due to their robustness and the simplicity of their error correction needs.

Deeper into Topological Quantum Computing

Now that you have a general introduction, this should provide a basis allowing the description of more details. In the following sections we will be detailing various aspects of topological quantum computing.

[1] https://news.microsoft.com/innovation-stories/azure-quantum-majorana-topological-qubit/

[2] https://arxiv.org/abs/1705.04103

Basics of the Quantum Hall Effect

As was discussed in the introduction, the quantum hall effect is critical to topological quantum computing. The quantum hall effect is related to both abelian and non-abelian anyons, and thus requires a bit more explanation. In a typical setup for observing the quantum hall effect, a thin layer of semiconductor material is cooled to very low temperatures and exposed to a strong perpendicular magnetic field. Under these conditions, the motion of electrons in the semiconductor becomes quantized into discrete energy levels known as Landau levels. These levels are a result of the quantization of the cyclotron orbits of electrons in a magnetic field.

This, of course, necessitates a bit more detail on Landau levels. Landau levels are quantized energy levels of charged particles, such as electrons, in a uniform magnetic field. This concept, introduced by physicist Lev Landau in 1930, is fundamental to understanding the quantum mechanics of electrons in magnetic fields and plays a crucial role in phenomena like the quantum Hall effects. When a charged particle moves in a magnetic field, it experiences a Lorentz force perpendicular to both its velocity and the magnetic field. This force causes the particle to move in circular orbits. In quantum mechanics, the energy of these circular orbits is quantized due to the constraints of the wavefunctions fitting into the circular path. These discrete energy levels are known as Landau levels.

When a current is passed through the material, electrons experience a force (Lorentz force) due to the magnetic field, which deflects them to one side of the conductor. This deflection leads to the buildup of a voltage perpendicular to both the current and the magnetic field, known as the Hall voltage. This produces the quantum hall effect, which leads to several interesting properties. The first being the quantized hall resistance. The Hall resistance R^{H} becomes quantized into plateaus at values given by $R_{\mathrm{H}} = \frac{h}{\mathrm{e}^2}\frac{1}{\nu}$, where h is the Planck constant, e is the elementary charge, and ν is the filling factor, an integer or fraction depending on the number of completely filled Landau levels and the nature of the effect (integer or fractional QHE).

We then come to the Integer Quantum Hall Effect and the Fractional Quantum Hall Effect. With the Integer Quantum Hall Effect, ν is an integer, and the quantization of the Hall resistance occurs because the electrons fill whole Landau levels. The IQHE can be explained using non-interacting electrons. The IQHE was discovered in 1980 by Klaus von Klitzing. This was mentioned earlier in this chapter. The Integer Quantum Hall Effect occurs when electrons confined to two dimensions at very low temperatures and subjected to a strong perpendicular magnetic field form discrete energy levels called Landau levels. At certain magnetic field strengths, the Hall conductance (conductivity) quantizes to integer values of $\frac{\mathrm{e}^2}{h}$, where e is the electron charge and h is Planck's constant.

The Fractional Quantum Hall Effect was discovered by Daniel Tsui and Horst Störmer in 1982. The FQHE appears at certain fractional filling factors. It arises from strong electron-electron interactions leading to the formation of new

quasiparticles with fractional charge, associated with complex many-body ground states. Unlike the IQHE, where the conductance steps are integer multiples, the FQHE shows quantized conductance at fractional values of $\frac{e^2}{h}$. Theoretical physicist Robert Laughlin proposed that the FQHE can be explained by the formation of a new quantum fluid state. The wavefunction of this state, known as the Laughlin wavefunction, describes the correlated behavior of electrons that minimizes their repulsive interaction energy, leading to the observed fractional quantization. These are now called Laughlin states.

Laughlin proposed a trial wavefunction to describe the ground state of the system at these fractional filling factors. For a filling factor $\nu = 1/m$ (where m is an odd integer), the Laughlin wavefunction is given by the following equation.

$$\Psi_m\left(z_1, z_{2,\ldots} z_n\right) = \prod_{i<j}\left(z_i - z_j\right)^m e^{-\frac{1}{4}\Sigma_k |z_k|^2}$$

where $z_i = x_i + iy_i$ are the complex coordinates of the i-th electron in a two-dimensional plane, and N is the number of electrons. Do not let the mathematics of this trouble you, if it is beyond your mathematical acumen. What is most important is to understand the concept of Laughlin states. Laughlin states are examples of topologically ordered states, where the order is characterized by global properties of the wavefunction rather than local order parameters. This concept is fundamental in understanding new phases of matter. The energy gap between the ground state and the excited states in the Laughlin state provides robustness against thermal fluctuations and impurities, leading to the observed quantization in the Hall conductance.

Abelian and Non-Abelian Anyons

Anyons are exotic quasiparticles that emerge in certain two-dimensional systems with an anyon is a specific type of quasiparticle that only occurs in two-dimensional systems. As you may imagine things are a bit different in two dimensions. Anyons have properties that make them distinct from the more familiar fermions and bosons. In quantum mechanics, particles are classified into two main groups based on their spin: fermions, which have half-integer spins, and bosons, with integer spins. In a two-dimensional world, two identical anyons change their wavefunction when they change places in ways that simply cannot happen in three-dimensional physics. They exhibit fractional statistics, meaning that when two anyons are exchanged, the wave function of the system acquires a phase that depends on the type of anyons and the order of exchange. Anyons can be broadly classified into two categories: Abelian anyons and non-Abelian anyons.

Anyons exhibit fractional statistics, which means their wave function acquires a phase that is a fraction of 2π when two anyons are exchanged. Anyons are often

protected by the topology of the underlying system, making them robust against local perturbations and noise. The manipulation of anyons through braiding, i.e., moving them around each other in a controlled manner, is a fundamental operation in topological quantum computation. Anyons can be used as qubits in quantum computation, with their states encoded in the topological properties of the system.

The concept of braiding anyons is central to their functionality. In a physical system, moving one anyon around another in a complete loop and then returning it to its original position can be thought of as "braiding." This action leads to changes in the quantum state that depend solely on the order and number of braiding operations, not on the details of how the braids are executed. The path taken by anyons during braiding forms a braid in spacetime. The outcome of anyon braiding depends only on the topology of the braid (the sequence and structure of crossings) and not on the specific details of the path. This topological nature makes the process robust against local errors and perturbations. The mathematical operations corresponding to braiding anyons are represented by unitary matrices that act on the Hilbert space of the system. For non-abelian anyons, these operators do not commute, leading to a rich structure for quantum computation.

Abelian Anyons

Abelian anyons behave similarly to conventional particles, such as fermions or bosons, in the sense that their exchange statistics follow either the Fermi-Dirac or Bose-Einstein statistics. Examples of systems that can host Abelian anyons include the fractional quantum Hall effect and certain types of topological superconductors. When two Abelian anyons are exchanged, the wave function of the system picks up a phase factor $e^{i\theta}$, where θ can be any value, not limited to 0 or $\pi\pi$ as with bosons or fermions, respectively. This phase factor is what defines their fractional statistics, as θ often corresponds to a fraction of π.

The statistical phase acquired during the exchange of anyons depends only on the topological class of their paths rather than the specific geometrical details. This topological aspect is critical because it implies that local disturbances or imperfections in the system have minimal impact on the exchange process, an important property for quantum computing.

The Abelian anyons are particularly interesting for quantum computing due to their robustness against local perturbations, courtesy of their topological nature. This makes them good candidates for topological quantum gates as well as quantum memory. Quantum gates based on the braiding of anyons can theoretically be implemented in such a way that they are inherently protected from certain types of errors. The operations depend only on the topology of the anyons' paths, making them less susceptible to the exact physical implementation errors. Storing information in the states defined by the braiding of anyons could potentially provide a form of quantum memory that is protected from local noise and errors.

Non-Abelian Anyons

Non-Abelian anyons are a more complex type of anyons that exhibit richer and more intricate quantum statistical behaviors compared to their Abelian counterparts. These anyons have generated significant interest due to their potential applications in topological quantum computing, where they could be used to implement quantum gates that are inherently protected against certain types of errors. The braiding of Abelian anyons can be used for topological quantum computation, although it is generally less powerful than the braiding of non-Abelian anyons. Non-Abelian anyons are more exotic and intriguing because their exchange statistics are noncommutative, meaning that the result of exchanging them depends not only on the order of exchange but also on the specific paths taken by the anyons. This noncommutativity enables topologically protected quantum information processing, making non-Abelian anyons particularly interesting for quantum computation. Examples of systems that may support non-Abelian anyons include certain types of fractional quantum Hall states and certain topological phases of matter.

Unlike Abelian anyons, where the exchange of two particles results in a simple phase factor, the exchange of non-abelian anyons can lead to a transformation of the quantum state into a different state within a multidimensional space of possible states. This property is due to the non-commutative nature of the operations involved in their manipulation. Non-Abelian anyons are associated with quantum states that are topologically protected. This means that the information encoded in these states is stored in the global properties of the system, making it robust against local disturbances. This topological protection is a key feature that makes these anyons highly valuable for quantum computing. The braiding (exchanging positions in specific patterns) of non-abelian anyons can be used to manipulate their quantum states. Because these states are multidimensional, the result of such braiding operations depends critically on the sequence and the nature of these operations, enabling the potential execution of complex quantum computations.

The study of anyons is a fascinating intersection of condensed matter physics, topology, and quantum information science. While Abelian anyons have been experimentally observed in systems like the fractional quantum Hall effect, the direct observation of non-Abelian anyons remains a topic of active research and experimentation. Their potential for fault-tolerant quantum computation makes them a subject of significant interest in the quest for scalable quantum technologies.

A promising candidate for realizing anyons in practical systems is the Majorana fermion, a type of quasi-particle that can emerge in certain superconducting materials. Majorana fermions are their own antiparticles and can exhibit non-abelian statistics, which are essential for the braiding operations in topological quantum computing. Majorana fermions were first proposed by the Italian physicist Ettore Majorana in 1937. This particle is intriguing because it is its own antiparticle, meaning that a Majorana fermion is identical to its own antiparticle. This concept sets it apart from more familiar particles like electrons and protons, which have distinct

antiparticles. Majorana's prediction was originally related to neutrinos, which are neutral particles involved in weak interactions. For a long time, it was an open question whether neutrinos were Majorana fermions, but more commonly, Majorana fermions have been discussed in terms of quasi-particles in solid-state systems rather than as fundamental particles.

Majorana fermions have been reported to be observed as emergent quasi-particles at the edges of topological superconductors or in semiconductor-superconductor nanowire interfaces under very specific conditions. These systems exhibit a pairing gap in their electronic structure, which can host Majorana modes at zero energy under the influence of a magnetic field and superconductivity, often realized via proximity to a conventional superconductor.

Fibonacci Anyons

Fibonacci anyons are quasi-particles or collective excitations that can emerge in two-dimensional systems under very specific conditions. They are named "Fibonacci" because their fusion rules follow patterns related to the Fibonacci sequence. Fibonacci anyons are special case of non-Abelian, meaning that when they are exchanged (braided), the quantum state of the system changes in a way that depends on the order of the exchanges. The fusion rules of Fibonacci anyons are particularly simple and powerful. If τ represents a Fibonacci anyon, then the fusion rules can be denoted as follows:

$$\tau \times \tau = 1 + \tau$$

Here, 1 represents the vacuum or trivial particle (essentially, no particle), and τ is another Fibonacci anyon. This means that when two Fibonacci anyons come together, they can either annihilate into the vacuum or form another Fibonacci anyon. These rules closely resemble the recursive nature of the Fibonacci sequence. In a quantum computing system based on Fibonacci anyons, the information is encoded in the state of the anyons, and quantum gates are implemented by braiding these anyons. The inherent fault tolerance of such systems comes from the fact that local perturbations do not easily affect the global topological properties determined by the braiding pattern, thus offering robust protection against errors.

Braids

A braid refers to a set of intertwined strands that can be described both visually and algebraically. The strands typically start at one set of points and end at another, and they are intertwined in such a way that no two strands cross each other more than once at any given point. Braids can be represented algebraically by braid groups.

The braid group on n strands, denoted B^n, consists of all possible braids with n strands, where two braids are considered equivalent if one can be transformed into the other without cutting the strands.

A braid group is a mathematical structure of n strands, forming a braid. In this case the anyons path through four-dimensional space time (also called a world line) form the braids. A braid group on n strands, denoted as B^n, consists of all possible braids involving n strands. The elements of B^n (the individual braids) can be combined (concatenated), which forms the group operation. The identity element of the group is the "trivial braid" where no strands are crossed, and each element (braid) has an inverse, which "undoes" the crossings. It is believed that such quantum computers would be more stable. In topological quantum computing, computational operations are performed by braiding anyons around each other. These braids are not just physical movements but are topological in nature, meaning that the paths encircled by the anyons, and their order of operations encode information and computation. Braid groups can be viewed topologically as the group of homeomorphisms (deformations) of a disk with n punctures (points removed), where the homeomorphisms fix the boundary of the disk. This view connects braid groups with the configurations of points in the plane and their movement paths.

Braid groups can be defined by generators and relations. For B_n, the generators are typically denoted as $\sigma_1,\sigma_2,\ldots,\sigma_{n-1}$, where each σ_i represents a specific crossing of the i-th strand over the $(i + 1)$-th strand. The relations between these generators reflect the ways in which the strands can be manipulated. Commutation relations are one type of relation. Commutation relationships $\sigma_i\sigma_j = \sigma_j\sigma_i$ for $|i - j| > 1$, meaning that crossings involving non-adjacent strands do not interfere with each other. Another relationship is the Braid relation, $\sigma_i\sigma_i + {}_1\sigma_i = \sigma_i + {}_1$, which captures the essential reordering possible within adjacent strands.

The process of adiabatically braiding these anyons around each-other will result in a unitary transformation which are gates. These braid operators are in two classes, the F matrix, or R matrix.

The R matrix is often thought of as the topological phase that is imparted onto the anyons during the braid. As the anyons wind around each-other, they acquire some phase due to the Aharonov–Bohm effect. The Aharonov-Bohm effect is a quantum mechanical phenomenon that reveals the fundamental aspect of electromagnetic potentials in quantum theory. Discovered by Yakir Aharonov and David Bohm in 1959, this effect shows that even in regions where magnetic fields are absent, the potential associated with a magnetic field can still influence quantum particles like electrons. This finding was pivotal because it challenged classical notions of how forces act in electromagnetism and deepened the understanding of quantum mechanics.

The F matrix is a result of actual physical rotations of the anyons. As they braid between each other, it is important to realize that the bottom two anyons, which constitute the control group, will still distinguish the state of the qubit. Therefore, braiding the anyons will alter which anyons are in the control group. Changing the control will change the basis. One evaluates the anyons by fusing the control group together first, so exchanging these anyons causes the system to rotate. Because these

anyons are non-abelian, the order of the anyons will matter, that order will transform the system.

Quantum gates in topological quantum computing are implemented by braiding anyons. The braiding process can be seen as performing a sequence of operations on the quantum information encoded in the anyons. The outcome of these operations depends only on the topological properties of the braid (the sequence and types of crossings) and not on the specific details of the paths taken, providing inherent error protection. Majorana zero modes in certain superconductors are examples of non-Abelian anyons that can be braided to perform quantum computations. The braiding of these modes is a practical approach being explored for building robust quantum computers.

Topological Field Theory

One will have a better understanding of topological quantum computing if one also understands topological field theory. The term *topological quantum field theory* refers to two related concepts. The first concept is that any quantum field theory in which the action is diffeomorphism invariant. One example of a diffeomorphism invariant is Chern-Simons theory. Chern-Simons theory is a field of study in theoretical mathematics and physics that emerged from the work of mathematicians Shiing-Shen Chern and James Harris Simons, who introduced the Chern-Simons form in the context of differential geometry in 1974. This theory plays a crucial role in various areas of modern theoretical physics, including quantum field theory, string theory, and condensed matter physics, particularly in the study of topological states of matter like quantum Hall effects and topological insulators.

The foundational element of Chern-Simons theory is the Chern-Simons 3-form, a secondary characteristic class used to construct invariants of three-dimensional manifolds. In simpler terms, it provides a way to assign numerical values (invariants) to certain geometric structures that remain unchanged under continuous deformations of these structures. In physics, Chern-Simons theory often refers to a type of gauge theory in three dimensions where the action (integral of the Lagrangian density over space and time) is given by the integral of a Chern-Simons 3-form. This is unique because, unlike conventional gauge theories which depend on metrics and detailed geometrical properties of spacetime, Chern-Simons theories are purely topological and do not depend on these metrics.

The second concept is any structure satisfying the Atiyah axioms. The two concepts are not unrelated. The matrix elements of the linear transformation corresponding to a cobordism are analogous to the transition amplitudes that one would compute by path integral in a conventional formulations of quantum field theory. The Atiyah axioms, formulated by the mathematician Sir Michael Atiyah, are a set of axioms that define a Topological Quantum Field Theory (TQFT). The axioms deal with n-dimensional manifolds and $(n + 1)$-dimensional cobordisms between them.

The axioms state that a TQFT assigns algebraic objects to these manifolds and cobordisms. Specifically, to each n-dimensional manifold, the TQFT assigns a vector space, and to each $(n + 1)$-dimensional cobordism, it assigns a linear map between vector spaces. Cobordism is a fundamental concept in topology, a branch of mathematics concerned with the properties of space that are preserved under continuous transformations. It is a way of classifying manifolds, which are mathematical spaces that locally resemble Euclidean space, based on the idea of morphing one manifold into another.

A manifold is a topological space that, at every point, resembles Euclidean space. Common examples include curves (one-dimensional manifolds), surfaces (two-dimensional manifolds), and higher-dimensional analogs. Two manifolds M and N of the same dimension are said to be cobordant if there exists a third manifold W such that the boundary of W is the disjoint union of M and N. In simpler terms, M and N can be thought of as the "ends" of W. This third manifold W is called a cobordism between M and N.

Research

There are numerous research papers published in the past few focused on topological quantum computing. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

An article titled "Topological Quantum Computation Assisted by Phase Transitions"[3] by Yuanjie Ren and Peter Shor delves into the integration of phase transitions in topological quantum computation (TQC). The reader will, of course, recognize Peter Shor's name as one of the luminaries of quantum computing.

The research proposes a novel approach to TQC by using anyon tunneling maps between sub-phases of the quantum double model D(G) for any arbitrary finite group G. The study extends the notion of Abelian Floquet codes to non-Abelian cases, increasing the capability and efficiency of topological quantum computing.

For those readers not familiar with Abelian Floquet codes, a brief discussion is provided here. Abelian Floquet codes are a subset of quantum error-correcting codes that utilize time-periodic systems to enhance quantum computing, particularly in the context of topological quantum computation. They form a part of a broader category of Floquet systems in quantum mechanics, where external controls or system parameters are varied periodically in time. These codes are derived from the Floquet theory of periodic Hamiltonians and represent an innovative approach to stabilizing quantum information against decoherence and operational errors. In quantum mechanics, Floquet theory deals with systems governed by Hamiltonians that are periodic in time. The solutions to the Schrödinger equation for such systems

[3] https://arxiv.org/abs/2311.00103

can be described using Floquet states, which are analogous to Bloch states in spatially periodic systems. Abelian Floquet codes are developed around time-periodic Hamiltonians that change cyclically, often structured to switch between different configurations or interaction patterns over discrete time steps.

The anyon tunneling map, denoted by ϕ, plays a crucial role in describing how anyons (quasiparticles with fractional quantum numbers) transition between different topological phases, facilitating more complex computational operations. The authors begin by reviewing the quantum double system and its derived phases, setting the foundation for understanding the complex interactions between different phases and sub-phases. They discuss the condensation processes and how anyons transition between different states, vital for realizing robust quantum computations.

The article "Progress in Topological Superconductors for Quantum Computing" highlights recent advancements made by researchers in developing topological superconductors aimed at enhancing the performance and stability of qubits for quantum computing. Researchers at Oak Ridge National Laboratory (ORNL) have created an ultra-thin superconductor by depositing specific elements and studying its topological properties. This work is directed towards improving the durability and functionality of qubits.

The team combined a superconductor, known for zero electrical resistance, with a topological insulator that has conductive surfaces but an insulating interior. This combination created a precise interface between the thin films, potentially leading to new physical phenomena beneficial for quantum computing. The ORNL team utilized advanced techniques such as angle-resolved photoemission spectroscopy (ARPES) and spin-resolved ARPES to study the electronic structure of the materials at the interface, enhancing their understanding of the interactions necessary for topological superconductivity.

Conclusions

Topological quantum computing has been primarily a theoretical approach to quantum information processing. However, this approach to quantum information processing has substantial advantages making it a worthwhile research area. There are also some practical applications of topological quantum computing.

Chapter 11
Neutral Atom-Based Quantum Computing

Introduction

Neutral atoms are atoms that have no net electric charge. In quantum computing, these atoms are typically cooled to extremely low temperatures, close to absolute zero, using laser cooling techniques. Each atom acts as a qubit, with quantum information encoded in the states of their electrons such as whether the electron is in the ground state or excited states. Quantum information is encoded in the internal states of the atoms, such as the hyperfine or Zeeman sublevels of their ground state. These states are manipulated using precision laser beams, which can change the energy state of an electron, thereby performing quantum logic operations.

A 2023 article in Physics world[1] discussed the fact that until recently, neutral atoms have not been a primary focus for quantum computing research. However, that does not mean that the topic is irrelevant to quantum hardware. This approach has potential and anyone studying quantum computing should be familiar with this topic.

Neutral atoms are individually trapped and manipulated using optical tweezers, which are tightly focused laser beams that can hold and move individual atoms. Arrays of optical tweezers can be dynamically reconfigured to arrange atoms in various patterns, essential for scaling up the system to many qubits. A key technology in neutral atom quantum computing involves exciting atoms to high-energy states known as Rydberg states, using lasers. Atoms in Rydberg states have exaggerated electromagnetic properties and can interact strongly with each other over relatively large distances, enabling the creation of entanglement between qubits. For those readers not familiar with Rydberg states, they are described in the following subsection.

[1] https://physicsworld.com/a/neutral-atom-quantum-computers-are-having-a-moment/

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_11

Rydberg States

Rydberg states refer to highly excited states of atoms or molecules in which one or more electrons are excited to a very high principal quantum number, n. These states are named after the Swedish physicist Johannes Rydberg. When an electron in an atom is in a Rydberg state, it is far from the nucleus and orbits in a region where the electrostatic potential from the nucleus is weak, resulting in some unique properties and behaviors. You will often see atoms in this state referred to as Rydberg atoms. Rydberg states are characterized by a high principal quantum number (n), typically much larger than the ground or low excited states (e.g., $n \geq 10$). As n increases, the size of the electron orbit also increases, and the electron behaves almost like a free particle.

Atoms in Rydberg states have significantly larger sizes compared to their ground state counterparts. The radius of the electron orbit scales as n^2, making these atoms tens of thousands of times larger than their normal size. The lifetimes of Rydberg states are relatively long, often in the microsecond to millisecond range. This is because the transitions between high-lying states involve low energy differences and hence emit photons less frequently. Rydberg atoms are highly sensitive to external electric and magnetic fields. This sensitivity increases with n because the electron is further from the nucleus and more susceptible to perturbations. One of the most notable properties of Rydberg atoms is their strong interactions with each other, mediated by dipole-dipole or van der Waals forces.

Quantum Operations

Entanglement is achieved by exploiting the strong interactions between Rydberg atoms. Quantum gates are implemented by precisely controlling the timing and pattern of laser pulses that excite atoms into and out of these Rydberg states. Neutral atom-based qubits tend to be scalable and to have long coherence times. However, the technology requires sophisticated apparatus to cool atoms to ultra-cold temperatures and to manipulate them with high precision using lasers.

Optical Tweezers

Optical tweezers were introduced earlier in this book and are a key component in neutral atom quantum computing. These are also sometimes called optical traps. Optical tweezers are a highly precise tool used to manipulate microscopic particles using laser light. The technology was developed by Arthur Ashkin, who won the 2018 Nobel Prize in physics for the development of optical tweezers. The basic principle behind optical tweezers is the use of light to exert force on objects. When

a laser beam is tightly focused, it creates a high-intensity light field. Particles near the focal point of the beam experience a force due to the gradient of the light's intensity. This force typically pulls the particle towards the region of highest light intensity, effectively trapping it at the focal point.

Gradient force is the primary mechanism at work in optical tweezers. Particles are attracted towards the area of highest intensity, usually the center of the beam, due to the gradient in the electric field of the light. In addition to the gradient force, there is also a scattering force that pushes the particle along the direction of the light beam. However, this force is generally weaker and is balanced by the gradient force to maintain the particle in the trap. A diagram of a generic optical tweezer is shown in Fig. 11.1.

The essential components are a laser source, which provides the coherent light needed for the trap. There is also usually a microscope which focuses the light to a fine point. High numerical aperture microscope objective lenses are used to focus the laser to a tight spot and to create a steep gradient in the light intensity. One component you may not recognize is the dichroic mirror. A dichroic mirror, also known as a dichroic filter or beam splitter, is a specialized mirror with the unique property of selectively passing light of a small range of colors while reflecting others. This is achieved through a process called thin-film interference, which involves multiple layers of thin films of dielectric material deposited on a glass or other transparent substrate. Dichroic mirrors are known for their precise control over which wavelengths are transmitted and which are reflected, making them highly effective in optical applications where such control is necessary.

Optical tweezers (or optical traps) can be single atom traps, or optical lattices that hold a number of neutral atoms in a regular pattern. Laser pulses can be used to change the internal states of trapped atoms, implementing single-qubit gates.

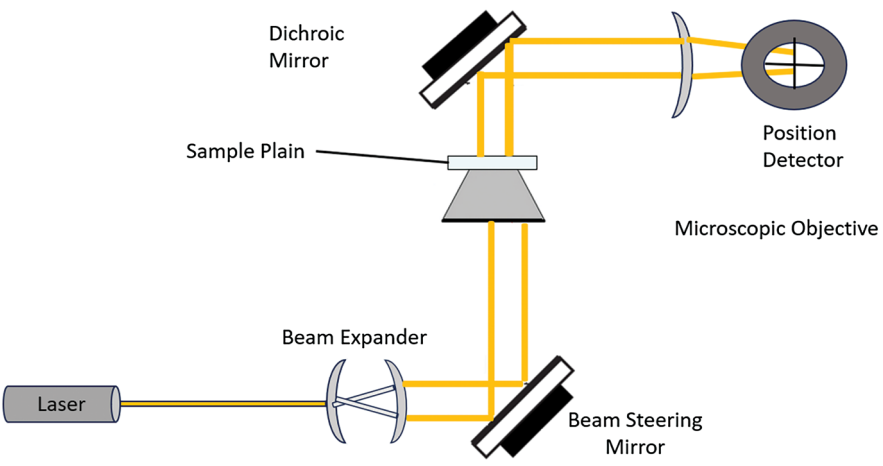


Fig. 11.1 Optical tweezer diagram

Controlled interactions between atoms in adjacent traps can be used to entangle qubits, enabling two-qubit gates.

Another option is to use magnetic traps. Magnetic fields can also be used to trap neutral atoms by exploiting the magnetic moments of the atoms' electron spins. Particles such as atoms with unpaired electron spins possess a magnetic moment, making them susceptible to magnetic fields. The interaction between the magnetic moment and an external magnetic field creates a force that can be used to trap the particle. A spatially varying magnetic field creates a gradient force on the magnetic moment of the particle, pulling it towards regions of lower or higher magnetic field strength, depending on the orientation of the magnetic moment.

There are several types of magnetic traps, some of which were discussed in Chap. 1 regarding ion traps and to some extent in Chap. 4 regarding Bose Einstein Condensate qubits. Quadrupole traps consist of two pairs of opposing magnetic coils arranged to create a field with a zero point in the center. The gradient of the magnetic field increases with distance from the center, providing a confining force for atoms with specific magnetic states. Another type of trap is the magnetic bottle which uses magnetic fields created by current-carrying coils or permanent magnets to form a three-dimensional trapping region.

Another type of trap that we have not discussed in previous chapters is the Ioffe-Pritchard Trap. Two or more coils along the axis of the trap generate a magnetic field that provides the axial confinement. The axial field ensures a minimum magnetic field strength at the trap center, preventing atoms from reaching a zero-field region. Figure 11.2 is a sketch of an Ioffe-Pritchard trap.

With an Ioffe-Pritchard trap additional coils at the ends of the trap provide further confinement and help shape the overall magnetic potential. Atoms are confined in a region where the magnetic field strength is non-zero and increases in all

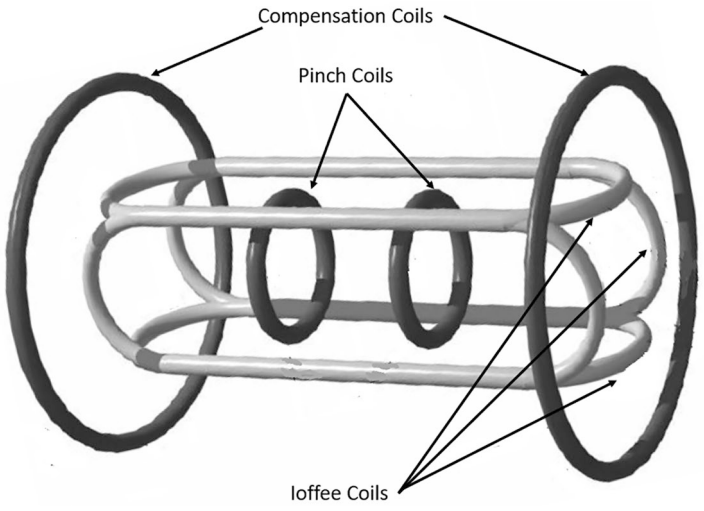


Fig. 11.2 Ioffee trap

directions away from the trap center. The axial bias field ensures that the magnetic field has a non-zero minimum value at the center of the trap.

Companies Using Neutral Atoms

Several companies are building quantum computers with neutral atoms. The following sub-sections will briefly describe the work being done by these companies.

Pasqal

Pasqal builds quantum computers using arrays of neutral atoms manipulated by optical tweezers. The company was founded by researchers from the Institut d'Optique in France. Like many quantum computing companies, Pasqal offers access to their quantum computing resources via cloud computing.[2] Pasqal uses arrays of neutral atoms as qubits. These atoms, typically rubidium, are cooled and trapped using optical tweezers. Each atom acts as a qubit, the fundamental unit of quantum information. Pascall has a timeline that they are hoping to make. That timeline is shown in Fig. 11.3 and hopes to reach 10,000 qubits by 2027.

Quera

Quera focuses on building quantum computers using arrays of neutral atoms in Rydberg states. The company arose from research at Harvard and MIT. As of this writing their flagship quantum computer is the 256 qubit Aquila. Aquila is also

Fig. 11.3 Pasqal timeline

[2] https://www.pasqal.com/solutions/our-solutions-and-offerings/

available via Amazon Braket.[3] Aquila uses neutral rubidium atoms as qubits. These atoms are cooled and manipulated using laser beams, allowing for precise control over their quantum states. The system employs optical tweezers to trap and arrange atoms in customizable configurations. Quera describes Aquill as a "field programmable qubit array" (FPQA).[4]

Quera provides a more detailed description of Aquill: "Aquila is a room-temperature quantum device with neutral Rb-87 atoms held and cooled to microkelvin temperatures by laser beams inside a vacuum cell. The individual atoms are intrinsically quantum, encoding qubits in their electronic states and manipulated by precisely controlled laser pulses. These states are used to process quantum information and can be detected by state-dependent fluorescence. The core of Aquila is a 2-cm scale glass vacuum cell, a microscope objective, and low-noise camera, surrounded by mostly off-the-shelf optical components delivering the laser beams to the atoms, and data-center style racks filled with control electronics for laser beams and data acquisition systems. The microscope objective focuses light into an area less than 200 μm wide—about the width of three human hairs—inside the glass cell, where the atoms themselves are arranged in a 2D pattern."

Atom Computing

Atom Computing[5] is another company focusing on neutral atom-based qubits. Atom has participated in the Defense Advanced Research Projects Agency (DARPA) Underexplored Systems for Utility Scale Quantum Computing (US2QC) program. The company currently has a 100-qubit prototype they have named Phoenix and are working on larger qubit implementations.

Research

There is a substantial body of research on the topic of neutral atom quantum computing. A sample of these papers is reviewed in this section. The interested reader should consider reading the original paper in its entirety for more details, but summaries of the papers are provided here.

A 2023 article titled "Neutral Atom Quantum Computing Hardware: Performance and End-User Perspective"[6] reviews the current state of neutral atom quantum

[3] https://www.quera.com/aquila

[4] https://assets-global.website-files.com/643b94c382e84463a9e52264/648f5bf4d19795aaf36204f7_Whitepaper%20June%2023.pdf

[5] https://atom-computing.com/

[6] https://epjquantumtechnology.springeropen.com/articles/10.1140/epjqt/s40507-023-00190-1

computing, focusing on hardware performance metrics relevant to industrial end-users. The article describes various quantum computing platforms, including superconductors, ion traps, neutral atoms, photons, and spins in semiconductors, highlighting the unique advantages and challenges of each. This comparative overview of physical qubits makes this an important article for anyone studying quantum computing.

The article's primary focus is on neutral atom quantum computing, outlining the properties that impact algorithm performance like qubit architecture, connectivity, gate fidelities, and stability. These factors are critical for both the quantum-part execution time and the end-to-end wall clock time. The article discusses specific benchmarks used to assess quantum computers, including low-level metrics (qubit count, stability, coherence, gate fidelity) and high-level system characteristics. The latter involves executing random unitaries and comparing predicted versus measured outcomes. The article provides an in-depth look at the physical foundations of neutral atom quantum computers, such as their architecture, the implementation of qubits, cooling and trapping atoms, and the significance of these hardware properties for practical applications.

A 2023 article in Nature[7] titled "High-fidelity parallel entangling gates on a neutral-atom quantum computer" focuses on advancements in neutral atom quantum computing, particularly in the area of entangling quantum gates. The researchers successfully implemented controlled-phase (CZ) gates with a fidelity of 99.5% across up to 60 neutral-atom qubits in parallel, which is above the threshold needed for quantum error correction using surface codes. This high fidelity was achieved by employing optimal control techniques, using atomic dark states to minimize errors from spontaneous scattering, and enhancing the Rydberg excitation and atom cooling methods.

The team focused on reducing errors from various sources such as spontaneous emission from intermediate atomic states and Rydberg-state decay. They employed a combination of advanced gate schemes based on numerical optimization to control these errors effectively. The entangling operations utilized Rydberg blockade interactions, facilitated by precisely arranged atoms in an optical tweezer array. This setup allowed for any-to-any qubit connectivity and highly parallel operations.

Conclusions

While the topic of neutral atom quantum computing has not been explored as substantially as some other physical qubit options, there have been advances in this area. In addition to companies actually using this approach, there is a growing body of research into neutral atom quantum computing.

[7] https://www.nature.com/articles/s41586-023-06481-y

Chapter 12
Reducing Noise and Error Correcting

Introduction

Quantum decoherence is a fundamental concept in quantum mechanics that describes the loss of quantum coherence in a system. In essence, it is the process by which a quantum system loses its ability to exhibit quantum behavior, such as superposition and entanglement, typically as a result of interacting with its environment in an uncontrollable way. This phenomenon is crucial for understanding the transition from quantum to classical behavior and is a key obstacle in the development and maintenance of quantum computers. There are issues regarding decoherence, noise, and error correction, all important for achieving practical, stable quantum computers.

Even if one could completely mitigate the issue of decoherence, error correction would still be an issue. Qubits are inherently probabilistic not deterministic. That fact alone will lead to a greater number of errors than one might encounter in classical computers. As you have read in other chapters in this book, some physical implementations of qubits are more error prone than others, requiring more robust error handling. Error handling also is more difficult with physical qubits because of their quantum behavior. The no-cloning theorem states that it is impossible to create an independent and identical copy of an arbitrary unknown state. This stems from earlier work showing that a non-disturbing measurement scheme simply cannot exist. A proof of the no-cloning theorem was published by Wooters and Zurek in 1982. One implication of the no-cloning theorem is that certain classical error correction methods, such as backup copies, cannot be used with quantum states.

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9_12

Decoherence

Theoretically, a quantum system could maintain coherence indefinitely if it were completely isolated from the external environment. But that is simply not possible as no system is perfectly isolated from its environment. There are many sources of interference from the external environment. It has even been demonstrated that cosmic rays and background environmental radiation can contribute to decoherence in quantum computers. Even entanglements can occur between the quantum computing system and its environment.

In addition to the external environment, the system itself can be an issue. For example, the physical equipment needed for things like implementing quantum gates, is constructed of atoms and particles that have quantum properties. The breadth of possible interference sources makes combating decoherence a rather difficult task. In general, many systems are only able to maintain coherence for a matter of microseconds to a few seconds.

Quantum mechanics allows a system to be in a superposition of many different states simultaneously. However, interaction with the environment effectively "measures" the quantum system, selecting out specific states over others based on the interaction. This selection breaks the superposition, forcing the system into one of the possible definite states. Superposition collapsing prior to measurement is a form of decoherence. As different parts of the quantum system interact differently with the environment, their phase relationships (which are critical for quantum coherence) get disrupted. This disruption leads to a loss of the coherent properties that allow for phenomena like interference. Phase disruption is another cause of disruption. Decoherence occurs when a quantum system is not perfectly isolated and starts interacting with its external environment (which can include everything from nearby particles to electromagnetic fields). These interactions cause the system to gradually lose its quantum characteristics and behave more classically.

One method used to help combat decoherence is super cooling to just a fraction of 1 K. In fact, some systems cool to a few nanokelvins. If you are not familiar, or do not recall the Kelvin scale, 0 K is the equivalent to -273.15 °C or -459.67 °F. This is not arbitrary. The point 0 K is absolute zero, which implies no thermal energy at all. Thus, nanokelvins are literally a fraction of a degree above absolute zero. To provide some context, temperatures in space are often around 2.7 K. Obviously, proximity to heat sources such as stars can alter that. The surface of Pluto plummets to about 40 K. So, quantum computing relies on temperatures that are much colder than Pluto, and even colder than deep space.

Helium-3 has a boiling point of 3.19 K.
Helium-4 has a boiling point of 4.214 K.
Liquid Helium is commonly used.

Intel uses cryogen-free dilution refrigerator systems from specialist Blufors[1] (A Finnish company specializing in supercooling). This works in stages cooling more at each stage. They use a mixture of Helium isotopes as the refrigerant. The Blufors cryogenic platform is shown in Fig. 12.1.

The reason supercooling is needed is expressed in this quote:

> At extremely cold temperatures, atoms and molecules simply move around less. Generally speaking, the lower a temperature is, the more stable a molecule becomes. Less movement means less energy being expelled. At a molecular level, which means that less energy is flying around, and consequently (since voltage and energy are directly related) less volatility in the voltage. This in turn means there is less of a chance that something outside of a human's control will cause a qubit's voltage to spike, causing the qubit to flip from one quantum state to another. Thus, keeping the computer cold introduces less energy into the system. This minimizes the chances of qubits incorrectly flipping in between quantum states.[2]

To give you some idea of temperatures actually used in quantum computing, consider temperatures of existing systems. As of October 2020, the IBM 50 qubit computer begins at about 800 mK and cools to 10 mK. D-Wave operates about 0.1833 K. As we have seen, these temperatures are colder than deep space. In 2020, researchers in Australia at the University of New South Wales were able to perform

Fig. 12.1 Blufors KIDE cryogenic platform

[1] https://bluefors.com/products/bluefors-products/

[2] https://www.qats.com/cms/2019/08/02/quantum-computing-cooling/

quantum computing at 1.5 K.[3] This was referred to as "Hot" qubits. As you can probably surmise, it takes substantial power to maintain such low temperatures. Also, in 2020, researchers developed quantum chips running at 1 K.[4]

Another method to mitigate decoherence is rapid pulse operations. This technique involves applying a sequence of rapid, controlled operations (pulses) to the quantum system to average out the effects of environmental noise over time. It is analogous to the spin echo techniques used in nuclear magnetic resonance. Closely related, regularly applying inversion pulses can refocus the system's quantum state, effectively filtering out low-frequency noise and preserving coherence.

Another interesting method for mitigating decoherence was proposed by a team of researchers in Israel. The researchers began by firing single photons at atoms and studying the results. When photons strike an atom, they are deflected. If the photon spin was not aligned with its path, then the photon and atom became entangled. However, if the photon spin was aligned with its path, it did not become entangled with the atom. This experiment suggests that creating some filtering mechanism for incoming radiation (i.e., light, cosmic rays, etc.) could substantially reduce the probability of those incoming particles becoming entangled with qubits and causing decoherence.

Noise

Closely related to decoherence is the concept of noise. In fact, some sources use noise and decoherence synonymously. As you can probably surmise, this is not a reference to actual audio signals. In a more general sense, noise is any issue that has a deleterious impact on computing results. This can be errors, faults, or yes decoherence.

Scientists at IBM have an interesting approach to the problem of noise. Their approach is counter intuitive. They have found that the use of a noise amplification technique helps improve the accuracy of a quantum computer. What this literally means is that they repeat a particular computation at varying levels of noise. This allows them to estimate, with high accuracy, what the result would have been in the complete absence of noise.[5,6]

Dynamic decoupling can also be used to protect qubits from environmental noise. It involves applying a sequence of pulses to the qubits that effectively average out the effects of certain types of noise. This is particularly useful for combating

[3] https://www.engadget.com/2018-01-09-this-is-what-a-50-qubit-quantum-computer-looks-like.html

[4] https://www.newscientist.com/article/2240539-quantum-computer-chips-demonstrated-at-the-highest-temperatures-ever/

[5] https://www.ibm.com/blogs/research/2019/03/noise-amplification-quantum-processors/

[6] https://venturebeat.com/2019/03/27/ibms-quantum-computation-technique-mitigates-noise-and-improves-accuracy/

dephasing, one of the most common types of noise in quantum systems. The basic principle of dynamic decoupling involves the application of a series of precisely timed pulses to the quantum system. These pulses are designed to refocus the quantum state and effectively average out the unwanted interactions between the qubits and their environment.

There are actually several methods or sequences of dynamic decoupling. One such method is the Uhrig Dynamical Decoupling (UDD) which arranges the timing of pulses according to the zeroes of Chebyshev polynomials. This pattern is highly efficient in protecting against pure dephasing noise and is optimized to handle a single, dominant noise frequency. Another is the Carr-Purcell-Meiboom-Gill (CPMG) sequence which consists of a series of equally spaced π pulses. It is particularly effective at correcting errors due to low-frequency noise and is used to extend the coherence time of a single qubit or an ensemble of qubits. Still another method is the Concatenated Dynamical Decoupling (CDD) which is a recursive application of dynamical decoupling, where a basic sequence is nested within itself to form a more robust decoupling strategy against a broader spectrum of noise frequencies.

Using environmental noise shielding is also an aspect of mitigating noise. This approach includes physical methods to reduce noise involve shielding the quantum computing apparatus from environmental factors that can cause decoherence, such as magnetic fields, temperature fluctuations, and electromagnetic interference. And of course, starting with physical qubits that have high coherence is also useful.

Error Correction

The quantum fault-tolerance theorem states that a quantum computer with a physical error rate below a particular threshold can suppress the logical error rate to arbitrarily low levels using quantum error correction. Worded slightly differently: if the physical qubits can keep error below a given level, the error correction algorithms can take it the rest of the way. There is some debate on what that threshold is, but at least down to 1% and some say 0.1%. There are some common methods for error correction that will be discussed in the following sections.

Shor Codes

Shor codes are one method of quantum error correction. Developed by Peter Shor, this was the first quantum error-correcting code. It uses nine qubits to correct arbitrary single-qubit errors, including bit-flip and phase-flip errors. The Shor code is a nine-qubit code, which means it encodes a single logical qubit into nine physical qubits. The primary purpose of the Shor code is to correct arbitrary single-qubit errors in a quantum system. One example is the bit-flip error, which can be thought of as comparable to classical bit errors where a 00 flips to a 11 and vice versa.

Single-qubit errors also include phase-flip errors, which affect the relative phase between the components of a quantum superposition.

The Shor code is essentially a combination of two classical error-correcting codes. The first is for bit-flip Errors. These are sometimes referred to as X errors. This part of the Shor code uses nine qubits to protect against bit-flip errors and is constructed by repeating the same bit three times, for each set of three qubits. For example, a logical $|0\rangle\rangle$ is encoded as $|000\rangle$and a logical $|1\rangle$ as $|111\rangle$. This repetition code can correct a single error in each block of three qubits. The second part of the error correction code is the code for phase-flip errors. Phase flip errors are sometimes referred to as Z errors. A phase flip changes $|0\rangle$ to $|0\rangle$ and $|1\rangle$ to $-|1\rangle\rangle$. The phase flip error correction is implemented after encoding for bit-flip errors, by applying a series of Hadamard transforms to the qubits. The Hadamard transform changes bit-flip errors into phase-flip errors and vice versa. The same structure that protects against one type of error pre-transformation protects against the other post-transformation. The encoding process for the Shor code can be described step-by-step as follows:

Step 1: Prepare the Initial State: Start with the quantum state you want to protect, denoted as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Step 2: First Layer of Encoding (Bit-flip Protection): Each qubit of the initial state $|\psi\rangle|\psi\rangle$ is encoded using a three-qubit repetition code. The state $|0\rangle|0\rangle$ is encoded as $|000\rangle|000\rangle$ and $|1\rangle|1\rangle$ as $|111\rangle|111\rangle$. Thus, the state $|\psi\rangle|\psi\rangle$ becomes $\alpha|000\rangle + \beta|111\rangle\alpha|000\rangle + \beta|111\rangle$.

Step 3: Second Layer of Encoding (Phase-flip Protection): Apply a Hadamard transform to each of the nine qubits. This step effectively prepares the system to be protected against phase errors.

Step 4: Entanglement of Blocks: Additional entangling gates (such as CNOT gates) are applied to entangle the blocks of three qubits, enhancing the code's ability to correct phase errors in addition to bit errors.

While there are many other error correction codes, which we will examine in the following subsections, the Shor code was the first quantum error correction code. That makes it important when studying quantum error codes.

Bacon-Shor Codes

The Bacon-Shor code is an example of a subsystem code. In quantum error correction, subsystem codes allow for a division of the system's Hilbert space into two parts: one part that holds the logical qubits, which is the information the code is attempting to protect, and another part that can be used for gauge qubits, which help in simplifying error correction procedures without directly affecting the stored quantum information. The code is constructed on a two-dimensional grid where qubits are arranged in rows and columns. Error syndromes (syndromes indicate the type and location of errors) are detected by measuring operators that involve only two-qubit interactions along rows and columns. This is less demanding in terms of

physical qubit connectivity compared to other codes that might require more complex multi-qubit interactions.

Decoding in the Bacon-Shor code involves simpler algorithms compared to some other quantum error-correcting codes. The structure of the code allows for efficient syndrome measurement and error correction procedures, which are vital for implementing quantum algorithms on noisy intermediate-scale quantum (NISQ) devices and beyond. Because of its reliance on only nearest-neighbor interactions and its ability to handle prevalent error types effectively. Therefore, the Bacon-Shor code is considered a commonly thought to be a strong choice for real-world quantum computing applications. It can be implemented in systems with limited qubit connectivity, such as superconducting qubits and trapped ions.

Stean Code

Steane code is an example of a CSS (Calderbank-Shor-Steane) code, it uses seven qubits and can correct any single-qubit error. The Steane code is based on classical Hamming codes, making it efficient for certain types of quantum hardware. The Steane code, also known as the seven-qubit code or CSS (Calderbank-Shor-Steane) code, is a prominent quantum error correction code that encodes one logical qubit into seven physical qubits. Developed by Andrew Steane in 1996, the code is based on classical error correction schemes and, like the Shore code, can correct both bit-flip and phase-flip errors. This code, like the Shor code, integrates ideas from two classical linear codes. It specifically uses the classical Hamming code, which encodes four classical bits into seven bits, allowing the detection and correction of a single bit error in each codeword.

The Steane code involves a construction that is symmetrical in handling both X (bit-flip) and Z (phase-flip) errors through the properties of the Hamming code. First this uses a classical Hamming code. The Hamming code can correct any single-bit error in a block of seven bits. It detects and corrects errors by using three parity check bits added to the four data bits, forming a codeword that is a combination of data and parity information. Secondly, the Steane code uses quantum encoding. To form the quantum Steane code, the classical Hamming code is used to correct bit-flip errors, and its dual code corrects phase-flip errors. The dual code has parity checks where the original has data bits and vice versa.

The process of the Steane code is shown here:

The process starts with the encoding process. In this process, the logical qubit is prepared as $\alpha|0\mathrm{L}\rangle + \beta|1\mathrm{L}\rangle$, where $|0\mathrm{L}\rangle|$ and $|1\mathrm{L}\rangle$ are encoded using the seven qubits based on superpositions of the codewords a.

$$\left|0\mathrm{L}\right\rangle = \frac{1}{\sqrt{8}}\left(\left|0000000\right\rangle + \left|1010101\right\rangle + \cdots + \left|0101010\right\rangle\right)$$

$$|1L\rangle = \frac{1}{\sqrt{8}}\left(|1111111\rangle + |0101010\rangle + \cdots + |1010101\rangle\right)$$

Ancilla qubits are used to perform syndrome measurements that diagnose whether a bit-flip or a phase-flip error has occurred and on which qubit. Bit flip errors are detected by measuring the parity of certain qubit subsets based on the classical Hamming code's parity checks. Phase flip errors are detected in a similar manner but require the system to be first manipulated (e.g., using Hadamard gates) to translate the phase-flip errors into bit-flip errors in the dual basis.

Implementing the Steane code in actual quantum systems requires precise control over seven qubits and the ability to perform complex quantum gates and syndrome measurements. While challenging, its properties make it a valuable tool in experimental quantum error correction, and it has been successfully implemented in various quantum computing platforms, including ion traps and superconducting qubits.

Raymond Laflamme Five-Qubit Codes

Canadian physicist Raymond Laflamme, working with colleagues Cesar Miquel, Juan Pablo Paz, and Wojciech Hubert Zurek developed a five-bit error correcting code. It should first be noted that Laflamme was a Ph.D. student of Stephen Hawking. The five-qubit code is designed to correct any single-qubit error in a block of five qubits. This includes errors due to both bit-flips (where a qubit state |0 > becomes |1 > and vice versa) and phase-flips (where the relative phase between |0 > and |1 > in a superposition state gets inverted), as well as a combination of both.

The code encodes a single logical qubit into five physical qubits. It is a stabilizer code, which means it is defined by a set of operators (called stabilizers) that commute with each other and whose simultaneous eigenstates define the code space. The stabilizers for the five-qubit code are generated from products of Pauli matrices. These stabilizers generate a stabilizer group under multiplication, and the code space is the simultaneous +1 eigenspace of these stabilizers.

For the five-qubit code, the stabilizers are given by the following products of Pauli matrices:

1. g1=X⊗Z⊗Z⊗X⊗I
2. g2=I⊗X⊗Z⊗Z⊗X
3. g3=X⊗I⊗X⊗Z⊗Z
4. g4=Z⊗X⊗I⊗X⊗Z

Here, X, Z, and I represent the Pauli-X, Pauli-Z, and the identity operators, respectively. These stabilizers commute with each other, which is a crucial property for any set of stabilizers. Logical operators for the encoded qubit, which effectively act on the logical qubit while leaving the encoded state within the code space, can

also be expressed in terms of tensor products of Pauli matrices. For the five-qubit code, the logical Pauli operators that act on the encoded logical qubit are:

Logical $X^{-}X^{-}$: $X^{-}=X\otimes X\otimes X\otimes X\otimes X$
Logical $Z^{-}Z^{-}$: $Z^{-}=Z\otimes Z\otimes Z\otimes Z\otimes Z$

These operators anti-commute with each other, just as the standard Pauli-X and Pauli-Z operators do for a single qubit, but they commute with all the stabilizers. The code corrects any single-qubit error, which includes bit-flip (X), phase-flip (Z), and both flips (Y where Y = iXZ). The error correction process involves the following steps:

Step 1: Syndrome Measurement: Measure the eigenvalues (+1 or −1) of the stabilizers g1, g2, g3, and g4. The combination of these eigenvalues (called the syndrome) determines the error that occurred.
Step 2: Error Diagnosis and Correction: Each syndrome uniquely corresponds to a specific error on one of the qubits or no error at all. Based on the measured syndrome, a specific correction operation (one of XX, ZZ, YY, or II) is applied to the appropriate qubit to restore the original logical state.

The Laflamme five-qubit code has a code distance of three, meaning it can correct any error affecting up to $\lfloor (d-1)/2 \rfloor$ qubits, where d is the code distance. In this case, $d = 3$ allows for single-qubit error correction. The code is also notable for being highly symmetric, which simplifies understanding and implementing it. With just five qubits, this code is the smallest possible code that can correct arbitrary single-qubit errors, which makes it highly efficient in terms of qubit overhead. The efficiency and minimalism of the code make it an exemplary model for theoretical studies and practical implementations in quantum computing.

Topological Codes

Topological codes are a class of quantum error-correcting codes that exploit the topology of the physical layout of qubits to protect quantum information against errors. These codes are highly regarded for their robustness and efficiency in error correction, making them a promising avenue for building scalable and fault-tolerant quantum computers. Two of the most well-known types of topological codes are Kitaev's toric code and the surface codes.

One of the seminal topological codes is Kitaev's toric code, developed by Alexei Kitaev. It is based on a two-dimensional lattice of qubits arranged on a torus (a doughnut-shaped surface).

The qubits are positioned on the edges of a square lattice that wraps around a torus. This lattice has periodic boundary conditions due to the torus's topology, meaning edges on one side of the lattice connect directly to the opposite side.

The toric code detects and corrects errors using stabilizer operators defined on the vertices (vertex operators) and faces (plaquette operators) of the lattice. First it will check for bit-flip errors by applying a Pauli X operator to each qubit surrounding a vertex. The product of these operations should ideally be $+1$; a -1 result indicates an error. Next it will check for phase-flip errors by applying a Pauli Z operator to each qubit around a plaquette (a cell in the grid). Similar to vertex operators, a -1 result indicates an error. The information is not stored in any specific qubit but in the state of the entire lattice. Logical qubits are defined by non-trivial loops around the torus, which can be manipulated without directly interacting with physical qubits, hence avoiding local errors.

Topological codes are considered one of the most viable strategies for realizing practical quantum computing, capable of operating under realistic noisy conditions. They are central to ongoing research in quantum error correction and are being actively explored in various experimental quantum computing platforms.

The second type of topological code is a surface code. Surface codes are popular due to their relatively high error thresholds and simpler error syndromes. The basic structure of a surface code involves arranging qubits on a two-dimensional grid. Each qubit is placed at the edges of a grid (often visualized as a checkerboard or similar lattice), and they interact with their nearest neighbors. The grid includes both data qubits, which hold the quantum information, and measurement qubits, which are used to detect errors.

Surface codes depend on two sets of qubits, the data qubits and the measurement qubits. As the name suggests, data qubits store quantum information. These are placed strategically across the lattice. The second set, the measurement qubits are interspersed with the data qubits and are used to measure error without disturbing the underlying quantum state. There are actually two types of measurement qubits:

- Vertex Operators (Z-type checks) which measure the product of the Pauli Z operators on surrounding data qubits.
- Face Operators (X-type checks) which measure the product of the Pauli X operators on surrounding data qubits.

Efficient implementation of surface codes requires that each qubit can interact with its nearest neighbors. This is more feasible in some quantum computing technologies (like superconducting qubits) than others. The surface code architecture is inherently scalable as adding more qubits enlarges the lattice, potentially increasing the robustness of the stored quantum information. Due to their robustness and relatively straightforward implementation requirements, surface codes are considered a leading approach for realizing scalable quantum computers. They are particularly well-suited for systems where local interactions between qubits are possible, and their high error thresholds make them practical under current technological limitations.

Floquet Error Correction Codes

Floquet error correction codes represent a novel approach in the field of quantum computing, aimed at protecting quantum information from errors using the principles derived from Floquet theory. Floquet theory, originally developed to describe systems under periodic driving, analyzes the behavior of quantum systems subjected to repeated periodic interactions. In classical and quantum mechanics, Floquet theory is used to study the behavior of systems that are subject to periodic driving forces. The theory provides a way to describe the evolution of these systems through what are known as Floquet modes and Floquet multipliers, which are the solutions to the time-dependent Schrödinger equation under periodic conditions.

The idea behind Floquet error correction codes is to exploit the periodic dynamics inherent in quantum systems influenced by external periodic controls (like microwave fields or laser pulses) to enact error correction. This approach is particularly relevant in systems where control operations can be naturally implemented in a periodic fashion. In a Floquet error correction code, the periodic driving is designed so that it continuously steers the quantum system back towards its intended state, effectively counteracting errors as they occur. This can be thought of as a dynamic way to maintain the coherence of quantum states without the need for interrupting the system to perform traditional error correction routines.

There are variations on the Floquet error correction codes. The Hastings-Haah Floquet code was proposed by Matthew Hastings and Jeongwan Haah, is designed to harness periodic dynamics to protect quantum information against errors. It employs the principles of Floquet theory, which deals with the behavior of systems under periodic driving, to create a new type of quantum error correction code that is particularly suitable for systems experiencing periodic changes or interventions. The central idea of the Hastings-Haah Floquet code is to encode quantum information in such a way that it naturally evolves back to its correct state through the periodic application of a specific sequence of gates. This method of error correction is dynamic and differs fundamentally from more traditional, static approaches where error correction routines are applied at discrete intervals.

Another variation is the Fracton Floquet error code. Fractons are a class of exotic quasi-particles that appear in certain strongly correlated quantum systems, characterized by their restricted mobility and unusual statistical behavior. Floquet engineering, on the other hand, involves controlling the properties of a quantum system through periodic driving, as described by Floquet theory. When these concepts are combined in the Fracton Floquet code, it offers a novel approach to quantum error correction with unique properties and challenges. Fracton topological orders are known for their subdimensional particle excitations, which means that these particles can only move along certain directions or planes within a material, or in some cases, not at all without assistance. This restricted mobility leads to highly localized error syndromes in quantum error correction codes, potentially simplifying the detection and correction of errors.

Majorana Stabilizer Code

The term *Majorana stabilizer code* refers to a class of quantum error correction codes that utilize Majorana fermions for storing and processing quantum information in a fault-tolerant manner. Majorana fermions in this context are considered as quasi-particles that appear in certain types of superconducting materials and have properties making them suitable for quantum computing, particularly in topological quantum computers. Majorana fermions were discussed in detail in Chap. 10.

Majorana stabilizer codes are built using Majorana operators, which are mathematical constructs that describe the quantum states of Majorana fermions. These operators are used to define the stabilizers in the code. Stabilizers are operators that commute with the Hamiltonian of the system and can fix the state of the system, thus preserving the quantum information encoded against certain types of errors. The non-Abelian statistics of Majorana fermions mean that their quantum state depends on the order in which they are interchanged or braided. This property allows for operations that are intrinsically protected from local disturbances. In Majorana stabilizer codes, these operations can help maintain the coherence of the quantum state, thus enabling more robust quantum error correction.

Gottesman-Kitaev-Preskill (GKP) Code

The Gottesman-Kitaev-Preskill (GKP) code is a type of quantum error correction code designed specifically for continuous-variable quantum systems. Eponymously named after its developers, Daniel Gottesman, Alexei Kitaev, and John Preskill, who introduced the code in 2001, the code is unique in its approach to encoding qubits in states of an oscillator, rather than in discrete two-level systems which are typical in other forms of quantum error correction. One example of an oscillator would be the quantum state of light. In the Gottesman-Kitaev-Preskill approach, logical qubits are encoded in the quantum states of an oscillator, typically using a superposition of states that form a regular grid in phase space (a plot of position vs. momentum). For example, a logical qubit 0 might be encoded by a superposition of position states at periodic intervals along the position axis, and a logical qubit 1 by a similar superposition shifted by half a period relative to the 0 state.

The Gottesman-Kitaev-Preskill code was created to correct for small shifts in the position and momentum of the oscillator. These shifts, in turn, correspond to common types of errors in continuous-variable systems. Examples of such common types of errors are those caused by thermal fluctuations or imperfections in control mechanisms. The Gottesman-Kitaev-Preskill code effectively quantizes these continuous errors by mapping them onto a discrete lattice in phase space, where they can be corrected by standard quantum error correction techniques.

Conclusions

There are numerous methods for combating decoherence and noise. Research and advances in these areas continue. Supercooling is one of the most common methods, but it is far from the only method. There are also a number of error correcting codes that can be used for quantum error correction. In this chapter, we have examined Shor codes, Stean codes, five-qubit codes, and topological codes. This should provide you with a good general understanding of quantum error correction.

Appendices

Appendix 1: Basic Quantum Computing

This book is written to focus on quantum hardware. As such, it assumes that you, the reader, have a basic working knowledge of quantum computing. However, we do not wish to leave any reader out. If you do not have a basic working knowledge of quantum computing, then you will find this appendix will provide you with sufficient knowledge to then follow the quantum hardware described in the chapters of this book.

Basic Quantum Physics

To best understand quantum physics, it is advantageous to learn the history of what led to quantum physics. There were several important discoveries in the latter days of the nineteenth century and the beginning of the twentieth century. Each of these discoveries added to a body of evidence that would eventually require the revision of classical physics and the birth of quantum physics.

The Nature of Light

One major issue that led to quantum physics, begins with understanding the nature of light. Is light a wave or a particle? The debate goes back to the 1600s. Isaac Newton had proposed that light was a particle or corpuscle. Christian Huygens had advocated that light was a wave, much like a sound wave or water wave. Christian Huygens was not alone in his view of light as a wave, but the prominence of Isaac Newton certainly gave weight to the corpuscular view of light. The debate regarding the nature of light would continue, until quantum physics.

C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9

These two competing concepts would continue to fuel debate in physics for quite some time until experiments could be conducted to help settle the matter. Waves generally propagate through some medium. Sound waves, as an example, propagate through the air. Proponents of the wave view of light proposed an *ether* that light propagated through. In fact, it was believed that all electromagnetic radiation propagated through this ether (also spelled aether). The concept of aether would dominate in physics until the very early twentieth century. This aether was thought to be a space-filling substance that was the transmission medium for not just light, but gravity and other forces. In fact, physicists attempted to understand planetary motion in the context of aether.

In 1801, Thomas Young performed an experiment that demonstrated the wave behavior of light. Young shined a light through two slits in a barrier and had a photosensitive plate on the other side. What occurs is that when the light went through two apertures, it appeared to have interference on the receiving side, much like a water wave or sound wave. Figure A.1 demonstrates these results.

Light is shown through the single slit, then on to the double slit. When the target is reached, the two light sources (one from each of the slits in the double slit) exhibit an interference pattern. The target will show stripes, which become wider as the target is moved further from the two slits. At the time, this seemed to settle the question of whether light was a particle or a wave. The light appeared to be clearly a wave, and not a particle, or corpuscle as the term was then used. Essentially, the thought was that if light was a particle, then one would expect the pattern on the opposite side to match the shape and size of the slit. But because an interference pattern was seen, this seemed to demonstrate light was a wave.

Further variations of the double slit experiment began to shed some light on the situation. First it was noted that sending waves/particles through the double slit one at a time, it certainly appeared as dots, or particles on the target. This can be done

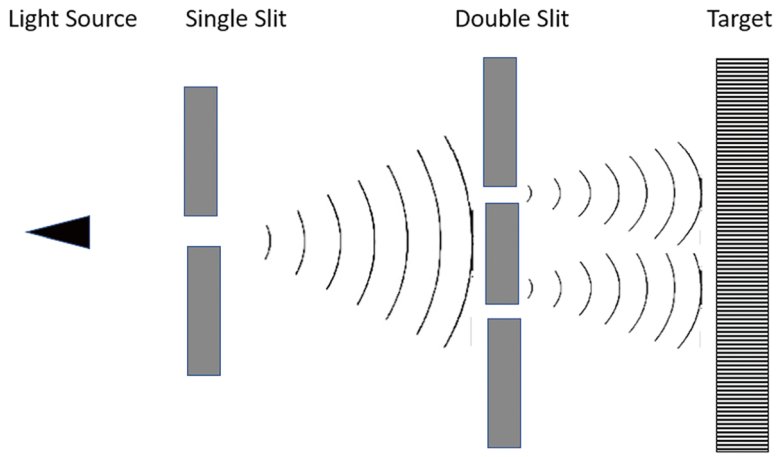


Fig. A.1 Double slit experiment

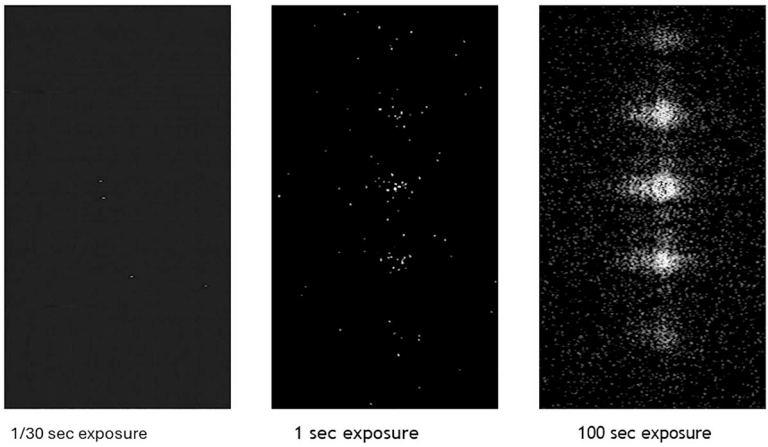


Fig. A.2 Using one slit

by simply closing one of the slits. But as more is sent through the slits, a wave pattern emerges. This is shown in Fig. A.2.

The next issue is the photoelectric effect. Essentially when some electromagnetic radiation, such as light, hits material, electrons are emitted (or other particles). This seems simple enough to understand. Classical (i.e., pre-quantum or relativity) electromagnetic theory held that the intensity of light would be proportional to the kinetic energy of the electrons emitted. This should mean that a dim light will produce lower energy electrons than a bright light. But experiments did not show this. Rather, experiments showed a correlation between frequency and the electrons dislodged. In fact, below a certain frequency, no electrons are emitted at all.

Einstein proposed this photoelectric effect was due to light being a collection of discrete wave packets, termed photons than a wave propagating through space. Work by Einstein, Plank, and others would eventually establish what is now known as wave-particle duality. This seems to counter the previous experiments by Young and others that clearly showed light was a wave.

Experiments of the double slit sort have been done with electrons, and other particles. There have been various experiments that demonstrated the wave behavior of matter. Among the most famous was the Davisson-Germer experiment. Without getting bogged down in all the details of that experiment, essentially in the 1920s, Clinton Davisson and Lester Germer showed that electrons scattered by the surface of a nickel-metal displayed a diffraction pattern, like a wave.

The concept of wave-particle duality is central to quantum physics. It is basically that a particle can be described as a particle or a wave. And that considering it as simply either a particle or wave is simply inaccurate. What we find is one set of data that seems to indicate light is a wave, and another set that seems to indicate that light is a particle. Essentially it leads to the idea that light is both a particle and a wave. This is also true for other subatomic particles. The concept is often referred to as *wave-particle duality*.

Blackbody Radiation

The history of quantum mechanics also includes some questions that may seem a bit obscure today. One of those issues was the study of black body radiation. This topic was very important in the 1800s. A black body is an opaque, non-reflective object. The radiation is thermal radiation (heat) in and around such a black body. A perfect absorber absorbs all electromagnetic radiation incident on it; such an object is called a black body. Obviously, such an object is hypothetical as no object can perfectly absorb energy.

When any object is heated, that heat has specific wavelengths. Those are inversely related to the intensity. As a body is heated and the temperature rises, the wavelength shortens; thus, the light changes colors. Measuring black body radiation led to results that did not seem to match existing theories. This was particularly true at higher frequencies. Balfour Stewert performed experiments comparing the radiation emitted from polished plates vs lamp black surfaces, using the same temperature for both. He used a galvanometer and a microscope to read the radiated energy. In his paper he did not mention wavelengths or thermodynamics. He claimed that heat radiated did so regardless of the surface being lamp black or reflective. Ultimately, he was in error.

In 1859, Gustav Kirchoff proved that the energy emitted from a black body depends only on the temperature of the black body and the frequency of the energy absorbed. He developed the rather simple formula in this equation.

Equation A.1: Kerchoff's Blackbody Energy

$$E = J\left(T, v\right) \tag{A.1}$$

In this formula, E is energy, T is temperature, and v is frequency. However, Kirchoff did not know what the function J was, and challenged other physicists to find it.

It has been established that the total power of the emitted radiation increases with temperature. This is known as Stefan's law, also known as the Stefan-Boltzmann law. This law describes the power radiated from a black body in terms of its temperature. Remember that a black body is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. The law states that the total energy radiated per unit surface area of a black body per unit time (j^*) is directly proportional to the fourth power of the black body's absolute temperature (T). Mathematically, this relationship is expressed as:

$$j^* = \sigma T 4$$

Here, j^* is the total power radiated per unit area (also known as the emissive power), T is the absolute temperature of the black body in kelvins (K), and σ is the Stefan-Boltzmann constant. The value of the Stefan-Boltzmann constant is approximately:

$$\sigma \approx 5.670374419 \times 10^{-8}\,\mathrm{W\,m^{-2}K^{-4}}$$

However, studies of black boxy radiation led to something called the ultraviolet catastrophe. The ultraviolet catastrophe, also known as the Rayleigh-Jeans catastrophe, refers to a historical problem in classical physics where theoretical predictions about black body radiation diverged drastically from experimental observations at short wavelengths (ultraviolet region). This discrepancy highlighted a fundamental flaw in classical physics and led to the development of quantum mechanics.

According to classical physics, specifically the Rayleigh-Jeans law, the energy density ($u(\lambda,T)$) of black body radiation at a wavelength λ and temperature T is given by:

$$u\left(\lambda,T\right) = \frac{8\pi k_{\mathrm{B}} T}{\lambda^4}$$

where k_{B} is the Boltzmann constant.

The Rayleigh-Jeans law suggests that as the wavelength λ decreases (towards the ultraviolet region), the energy density $u(\lambda,T)$ increases without bound, implying that the total emitted power would be infinite for short wavelengths. That is the ultraviolet catastrophe, obviously infinite emitted power is just not possible.

Experiments showed that the energy density increases with decreasing wavelength up to a certain point, after which it drops off instead of continuing to rise infinitely. This contradicted the Rayleigh-Jeans prediction and was especially noticeable at short (ultraviolet) wavelengths.

In 1900, Max Planck proposed a solution by introducing the idea that electromagnetic radiation is quantized. He suggested that energy is emitted or absorbed in discrete units (quanta) proportional to the frequency (ν) of the radiation:

$$E = h\nu$$

where h is Planck's constant.

By incorporating this quantum hypothesis, Planck derived a new formula for black body radiation that accurately matched experimental results across all wavelengths:

$$u\left(\lambda,T\right) = \frac{8\pi hC}{\lambda^5}\left(\frac{1}{e^{\dfrac{hC}{\lambda k_{\mathrm{B}} T}} - 1}\right)$$

where c is the speed of light.

Planck's law resolved the ultraviolet catastrophe by predicting that the energy density would peak at a certain wavelength and then decrease, consistent with observations. It also showed that things at the quantum level are "quantized" rather than contiguous. Thus, the name, quantum physics

Entanglement

This particular facet of quantum physics may be the oddest yet. In fact, Albert Einstein famously called this "spooky action at a distance." In this section, I will not be able to provide you with the "why" of entanglement. That is because we simply do not know. Whoever can fully explain this is certainly a candidate for a Nobel prize. However, countless experiments have confirmed it is true.

Entanglement begins with two particles being generated in such a way that the quantum state of each particle is inextricably linked to the other. Before we continue this description, it is very important that you realize that this is not some theoretical concept. Entanglement has been rigorously verified in countless independent experiments.

If one particle in an entangled pair has a spin that is clockwise, the other particle in the pair will have a counterclockwise spin. A 1935 paper by Albert Einstein, Boris Podolsky, and Nathan Rosen described this process as what would become known as the EPR paradox. Einstein insisted that this meant there must be some issue with our understanding of quantum physics. The EPR paradox involves a thought experiment. In the experiment, one considers a pair of entangled particles. The issue in the paper was the if these particles are so entangled that their states are complementary, then altering the state of the first particle could instantaneously change the state of the other particle. This would, essentially, mean information regarding the state of the first particle was transmitted instantly to the second particle. The theory of relativity tells us that nothing can travel faster than the speed of light. In the alternative, the state of the second particle must be set before any measurement of either particle is taken. And this was counter to both theory and experiment, which demonstrates that particles are in a state of superposition until measured, and then the wave function collapses into a single state. Thus, a paradox exists.

Essentially entangled particles are treated not as individual particles but rather as a whole. The state of this composite system is the sum or superposition of the product states of each local constituent. There are various ways particles become entangled. Subatomic particle decay is one such way. What we find consistently is that if we separate two entangled particles, then measure a property of one particle, such as spin, then measure the other particle's property, the two will be complementary. They are inextricably entangled.

There have been several different proposed explanations for entanglement. None have been verified. It is beyond the scope of this appendix or this book to more fully explore entanglement. But keep in mind that the fact of entanglement has been experimentally verified numerous times. However, we can briefly review some of the proposed explanations, even those that may have fallen out of favor in the physics community.

The hidden variables hypothesis contends that the particles actually have some hidden variables that, right at the moment the particles are separated, determining the outcome of properties such as spin. This would mean that there really is no

non-locality, simply variables we do not know about. Einstein was a proponent of this idea. However, no experiments have given any evidence to support this.

It is also known that entanglement occurs naturally. For example, in multi-electron atoms, the electron shells always consist of entangled electrons. So, entanglement is not just the result of physicists experimenting with nature. It is how nature actually works.

Qubits

A qubit is simply storing data in some observable of some particle (electron, photon, etc.). An observable is some property we can measure such as polarization, energy, etc. A qubit is a two-state quantum-mechanical system. Spin or polarization work well. The qubit, unlike a bit, need not be in one state or the other, but is in a superposition of states. This is often represented with what is called a Bloch sphere, shown in Fig. A.3.

There are two primary operations that can be performed on a qubit. The first is measurement. This is usually more formally called standard basis measurement. This is an operation that will provide a measurement that will either result in a $|0>$ or a $|1>$ based on the aforementioned probabilities $|\alpha|^2$ and $|\beta|^2$. It is critical to realize that with qubits, unlike classical bits, the measurement is an irreversible operation. Remember, before the measurement the qubit is in a state of superposition. After measurement that superposition collapses to one of the two basis states.

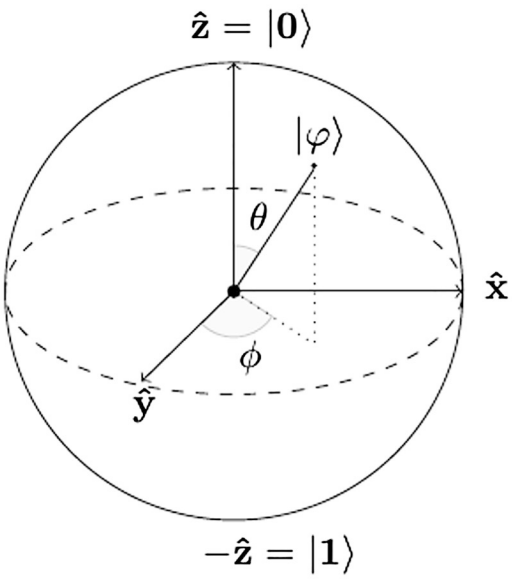


Fig. A.3 The Bloch sphere

Bra-Ket Notation

States of a qubit can be represented as either a column matrix or using Bra-Ket notation. This is typically written as

$$|0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1> = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Bra-ket notation also called Dirac notation is used to represent vectors. In this case the two states |0> and |1> are both kets. If one wishes to represent two qubits, then there are four states to be represented:

$$|00> = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } |11> = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } |01> \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and} |10> \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Keep in mind that these are the states you will have once you have measured the qubit. Until such measurement occurs, the qubit is in a superposition of possible states. Therefore, to describe a qubit the following formula is useful.

$$|\psi> = \alpha |0> + \beta |1>$$

The values α and β are the probability amplitudes. Usually, these probabilities are complex numbers. This is an important concept. Until measured, or the wave function otherwise collapses, it is not simply that we do not know the state of the particle, it is that it is in a superposition of possible states with only a probability that it will collapse to a specific state. This probabilistic nature of qubits makes them ideal for certain problems, such as optimization problems, but wholly unsuited for other problems.

Born's Law

To understand Born's law, let us begin with some observable. That observable corresponds to a self-adjoint operator A. A has a discrete spectrum measured in a system and the system has a normalized wave function. Born's law tells us that given these circumstances, we know some specific things about this system. If we have an observable corresponding to some self-adjoint operator, we will call A with a discrete spectrum is measured in a system that has a normalized wave function, there are several things we know. However, before we get too what Born's rule tells us (note, that I have intentionally used both "Born's rule" and "Born's law"), you must understand what a self-adjoint operator is. Given some complex vector space (think back to lesson 1) a self-adjoint operator is a linear map from V to itself. In another words, it is its own adjoint. Sometimes you may see the adjoint called a Hermitian conjugate.

The Born's rule (sometimes called Born's postulate or Born's law) provides us with the probability that a measurement of a quantum system (such as a qubit) will yield a particular result. This was formulated by Max Born in 1926. Among other things that Born's rule teaches us is that it states that the probability density of finding a particle at a particular point is proportional to the square of the magnitude of the particle's wavefunction at that point. Born's rule tells us the probability of outcome |0> with value '0' is $|\alpha|^2$ and the probability of the outcome being |1> with the value '1' is $|\beta|^2$. Remember α and β are the probability amplitudes and are going to be complex numbers. Given that probabilities always equal to 1, we can say that

$$|\alpha|^2 + |\beta|^2 = 1.$$

Quantum Gates

In quantum mechanics the quantum state can evolve in two ways, by Schrödinger's equation (unitary transformations), or by the collapse of their wave function. Logic operations for quantum computers are unitary transformations and therefore evolve reversibly. Remember that a unitary transformation is one that preserves the inner product. More Formally, a unitary transformation is an isomorphism between two Hilbert spaces. It is therefore a bijective function. An isomorphism is a structure-preserving mapping between two structures of the same type that can be reversed by an inverse mapping. The quantum logic gates are reversible unitary transformations on at least one qubit. Quantum logic gates, unlike classical logic gates, are all reversible. As with classical gates, the quantum gates take input and produce some output. These are operations on qubits.

Many of the gates used in classical computing are not reversible. The logical AND gate is an example of a non-reversible classical gate. The NOT gate is a classical gate that is reversible. In general, a gate is said to be reversible if the input vector can be uniquely recovered from the output vector and there is a one-to-one correspondence between its input and output assignments. This means that not only can the outputs be uniquely determined from the inputs, but also the inputs can be recovered from the outputs.

Reversible gates are necessary for effective computing. This is true of classical computers and quantum computers. A more mathematically rigorous description of classical reversible gates is given here: an n-bit reversible gate is a bijective mapping f from the set $\{0,1\}^n$ of n-bit data onto itself. For some readers, the term *bijective* may be new. There are actually three related terms that need to be defined. Those terms are injective, surjective, and bijective. An easy way to understand these terms is to consider two sets of points, we will call A and B. In its simplest terms, an injection means that everything in A can be mapped to something in B, but there might be items in B with nothing matching in A. Surjective means that every B has at least one matching A, but could actually have more than one match in A. Bijective means every A has a B and vice versa. Put more formally; it means both injective and surjective. This is shown in Fig. A.4.

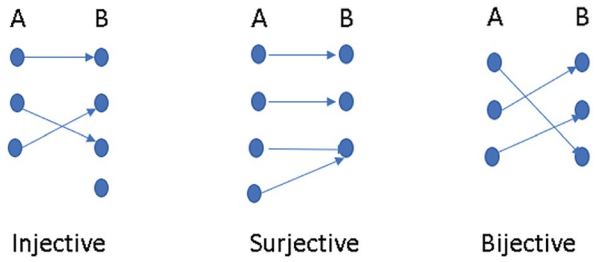


Fig. A.4 Bijective-surjective-injective

Bijective functions are invertible. This will be important when we discuss reversible gates and circuits. Unitary matrices represent quantum logic gates.

Perhaps the most widely discussed quantum gate is the Hadamard gate. It is represented by the Hadamard matrix, shown here:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

It is named after the French mathematician Jacques Hadamard. Note the Hadamard gate multiplies the Hadamard matrix by $\frac{1}{\sqrt{2}}$. Returning to the matrix itself, you may note that the rows are mutually orthogonal. The Hadamard gate operates on a single qubit.

The basics of Pauli matrices are not difficult. The "why" of Pauli matrices might be just a bit more complicated. These are a set of three 2 × 2 matrices. The three Pauli matrices are:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These are usually represented with the σ notation, thus:

$$\sigma 1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma 2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma 3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The representation of these three matrices is certainly not complicated. But they have two properties. These matrices are both Hermitian and unitary. Hermitian refers to a square matrix that is equal to its own conjugate transpose. Conjugate transpose means first taking the transpose of the matrix, then taking the complex conjugate of the matrix. The symbol for transpose is

$$\mathrm{T}: \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathrm{T} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}.$$

Let us look at one of the Pauli matrices to see this in a more concrete form:

$$\sigma 1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathrm{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

That demonstrates the transposition, but where does the conjugate come in? A square matrix is unitary if its conjugate transpose is also its inverse. Put in another way, a unitary matrix is a complex generalization of an orthogonal matrix. Unitary matrices are characterized by the following property, for a matrix A:

$$\mathrm{A}\mathrm{A}^{\mathrm{T}} = \mathrm{A}^{\mathrm{T}}\mathrm{A} = \mathrm{I}$$

Consider the following Pauli matrix:

$$\sigma 2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

This could be represented as:

$$\sigma 2 = \begin{bmatrix} 0+0i & 0-i \\ 0+i & 0+0i \end{bmatrix}$$

Well, if you take the complex conjugate (simply changing the sign for the imaginary portion) you get:

$$\sigma 2 = \begin{bmatrix} 0-0i & 0+i \\ 0-i & 0+0i \end{bmatrix}$$

Now if you take the transpose of that:

$$\begin{bmatrix} 0-0i & 0+i \\ 0-i & 0+0i \end{bmatrix} \mathrm{T} = \begin{bmatrix} 0-0i & 0-i \\ 0+i & 0+0i \end{bmatrix}$$

Which simplifies to:

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

The matrix is unchanged. The basic mechanics of these matrices do not seem too difficult. Now let us look a bit more into the why. These three matrices are named after the physicist Wolfgang Pauli. These occur in the Pauli equation which takes into account the spin of a particle with an external electromagnetic field. The Pauli-X gate is the quantum equivalent of the classical NOT gate. It is the equivalent of a rotation around the X-axis of the Block sphere by π radians. It will transform |0> to |1> and vice versa.

Phase shift gates operating on a single qubit and leave the basis state |0> unchanged but map the basis state |1> to $e^{i\phi}$|1>. Interestingly, phase shift gates do not change the probability of measuring a |0> or |1> after they are applied. These gates, instead, modify the phase of the quantum state, thus the name. A generalized phase shift gate is shown here:

$$R_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

Phase shift gates are used in many places, but specifically in transmons. A transmon is a superconducting charged qubit, which was designed to have reduced sensitivity to interference.

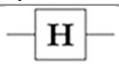
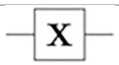
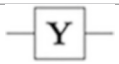
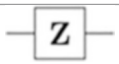
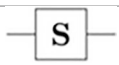
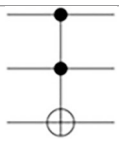
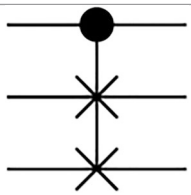
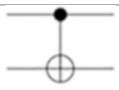
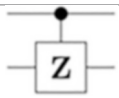
While there are other forms of controlled gates, the general class of controlled gates operating on qubits using specific operations, are widely used. For example, the controlled NOT gate (often called the cNOT or cX gate) acts on 2 qubits. If the first qubit is |1> then the NOT operation is performed on the second qubit. If the first qubit is not |1> then the second qubit is unchanged.

Table A.1 shows symbols used in describing quantum circuits.

Summary

Obviously, a single appendix cannot provide you with expertise in basic quantum physics or quantum computers. However, the goal is to provide the reader just enough information to be able to follow what is presented in the chapters of this book. It is hoped that this appendix, along with Appendix 2 for those who need it, will provide that basic level of information.

Table A.1 Quantum circuit symbols

Gate	Symbol
Hadamard	
Pauli-X	
Pauli-Y	
Pauli-Z	
SWAP	
Phase-shift	
Toffoli	
Fredkin	
Controlled	ΛG
Controlled s (CNOT, CX)	
Controlled Z	

Appendix 2: Mathematics for Quantum Computing

Linear Algebra

Linear algebra was initially developed as a method to solve systems of linear equations (thus the name). However, we will move beyond solving linear equations. Linear equations are those for which all elements are of the first power. Thus, the following three equations are linear equations:

$$a + b + c = 74$$

$$3x + 2y = 33$$

$$2x + y - z = 21$$

However, the following are not linear equations:

$$5x^2 + 3 = 11$$

$$4y^2 + 6x + 2 = 22$$

$$7x^4 + 2y^3 - 1 = 7$$

The first three equations have all individual elements only raised to the first power (often that with a number such as x^1 the 1 is simply assumed and not written). But in the second set of equations, at least one element is raised to some power greater than 1. Thus, they are not linear. While linear algebra was created to solve linear equations, the subject has grown to encompass a number of mathematical endeavors that are not directly focused on linear equations. The reason for that is that linear algebra represents numbers in matrix form, which we shall examine in some detail in this chapter.

It may help to begin with a very brief history of linear algebra. One of the earliest books on the topic of linear algebra was "Theory of Extension" written in 1844 by Hermann Grassman. The book included other topics, but also had some fundamental concepts of linear algebra. As time progressed, other mathematicians added additional features to linear algebra. In 1856, Arthur Cayle introduced matrix multiplication. Matrix multiplication is one of the elements of linear algebra that has applications beyond solving linear equations. In 1888, Giuseppe Peano gave a precise definition of a vector space. Vector spaces also have applications well beyond solving linear equations. Linear algebra continued to evolve over time.

It also was a bit of a surprise to physicist to find out that linear algebra was an excellent way of describing quantum states.

Linear Algebra Basics

Quantum physics and quantum computing frequently deals with matrices and vectors. Therefore, it is appropriate to begin this exposition of linear algebra with a discussion of matrices. A matrix is a rectangular arrangement of numbers in rows and columns. Rows run horizontally and columns run vertically. The dimensions of a matrix are stated m x n where m is the number of rows and n is the number of columns. Here is an example:

$$\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$$

If this definition seems a bit elementary to you, you are correct. This is a rather straightforward way to represent numbers. The basic concept of a matrix is not at all difficult to grasp. That is yet another reason it is an appropriate place to begin exploring linear algebra. A matrix is just an array that is arranged in columns and rows. Vectors are simply matrices that have one column or one row. The examples in this section focus on 2 × 2 matrices, but a matrix can be of any number of rows and columns, it need not be a square. A vector can be considered a 1 × *m* matrix A vector that is vertical is called a column vector, one that is horizontal is called a row vector. Matrices are usually labeled based on column and row:

$$\begin{bmatrix} a_{ij} & a_{ij} \\ a_{ij} & a_{ij} \end{bmatrix}$$

The letter *i* represents the row, and the letter *j* represents the column. A more concrete example is shown here:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

This notation is commonly used for matrices including row and column vectors. In addition to understanding matrix notation, there are some common types of matrices you should be familiar with. The most common are listed here:

Column matrix: A matrix with only one column.
Row matrix: A matrix with only one row.
Square matrix: A matrix that has the same number of rows and columns.
Equal matrices: Two matrices are considered equal if they have the same number of rows and columns (the same dimensions) and all their corresponding elements are exactly the same.
Zero matrix: Contains all zeros.

Matrix Addition and Multiplication

Addition and multiplication of matrices are actually not terribly complicated. There are some basic rules you need to be aware to determine if two matrices can be added or multiplied. If two matrices are of the same size, then they can be added to each other by simply adding each element together. You start with the first row and first column in the first matrix and add that to the first row and the first column of the second matrix thus in the sum matrix. This is shown in Eq. B.1.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \mathrm{A}_{11} + b_{11} & a_{12} + b_{12} \\ \mathrm{A}_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} \tag{B.1}$$

Consider the following more concrete example given in Eq. B.2.

$$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} \tag{B.2}$$

This is rather trivial to understand, thus this journey into linear algebra begins with an easily digestible concept. Multiplication, however, is Somewhat more difficult. One can only multiply two matrices if the number of columns in the first matrix is equal to the number of rows in the second matrix. First let us take a look at multiplying a matrix by a scalar (i.e., a single number). You simply multiply the scalar value by each element in the matrix as shown in Eq. B.3.

$$C\begin{bmatrix} a_{ij} & a_{ij} \\ a_{ij} & a_{ij} \end{bmatrix} = \begin{bmatrix} ca_{ij} & ca_{ij} \\ ca_{ij} & ca_{ij} \end{bmatrix} \tag{B.3}$$

For a more concrete example, consider Eq. B.4.

$$3\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 6 & 9 \end{bmatrix} \tag{B.4}$$

As you can observe for yourself, the addition of two matrices is no more complicated than the addition you learned in primary school. However, the multiplication of two matrices is a bit more complex. The two matrices need not be of the same size. The requirement is that number of columns in the first matrix is equal to the number of rows in the second matrix. If that is the case, then you multiply each element in the first row of the first matrix, by each element in the second matrixes first column. Then you multiply each element of the second row of the first matrix by each element of the second matrixes second column. Let us first examine this using variables rather than actual numbers. This example also uses square matrices to make the situation even simpler. This is shown in Eq. B.5.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \tag{B.5}$$

This is multiplied in the following manner:

$$\begin{aligned} a^*e + b^*g &\quad \left(a_{11}{}^*b_{11} + a_{12}{}^*b_{21}\right) \\ a^*f + b^*h &\quad \left(a_{11}{}^*b_{12} + a_{12}{}^*b_{22}\right) \\ c^*e + d^*g &\quad \left(a_{11}{}^*b_{11} + a_{12}{}^*b_{21}\right) \\ c^*f + d^*h &\quad \left(a_{11}{}^*b_{11} + a_{12}{}^*b_{21}\right) \end{aligned}$$

Thus, the product will be

$$\begin{matrix}(a*e+b*g) & (a*f+b*h)\\ (c*e+d*g) & (c*f+d*h)\end{matrix}$$

It is worthwhile to memorize this process. Now, consider this implemented with a concrete example, as shown in Eq. B.6.

$$\begin{bmatrix}1 & 3\\ 3 & 1\end{bmatrix}\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix} \tag{B.6}$$

We begin with:

$$\begin{aligned}1^{*}2+3^{*}1&=5\\ 1^{*}2+3^{*}3&=11\\ 3^{*}2+1^{*}1&=7\\ 3^{*}2+1^{*}3&=9\end{aligned}$$

The final answer is:

$$\begin{bmatrix}5 & 11\\ 7 & 9\end{bmatrix}$$

It should first be noted that one can only multiply two matrices if the number of columns in the first matrix equals the number of rows in the second matrix. That should be clear if you reflect on how the multiplication is done. If you keep that rule in mind, then the multiplication is not particularly complicated, it is simply tedious.

It is important to remember that matrix multiplication, unlike multiplication of scalers, is not commutative. For those readers who may not recall, the commutative property states $a^{*}\,b = b^{*}\,a$. If a and b are scalar values then this is true, regardless of the scaler values (e.g., integers, rational numbers, real numbers, etc.). However, when multiplying two matrices this is not the case. This is often a bit difficult for those new to matrix mathematics. However, it is quite easy to demonstrate that this commutative property does not hold for matrices. For example, consider the matrix multiplication shown in Eq. B.7.

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}8 & 5\\ 13 & 8\end{bmatrix} \tag{B.7}$$

However, if you reverse that order you will get a different answer. This is shown in Eq. B.8.

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 7 \\ 7 & 4 \end{bmatrix} \tag{B.8}$$

Seeing an actual example, it becomes immediately apparent that matrix multiplication is not commutative. You may find some instance that just incidentally appears communitive. However, that is not sufficient. To have the commutative property, it must be the case that regardless of the operands chosen, the result is commutative.

Other Matrix Operations

Matrix addition and multiplication are perhaps the most obvious of operations on matrices. These operations are familiar to you from previous mathematics you have studied. However, there are some operations that are specific to matrices. Matrix transposition is one such operation. Transposition simply reverses the order of rows and columns. While the focus so far has been on 2 × 2 matrices, the transposition operation is most easily seen with a matrix that has a different number of rows and columns. Consider the matrix shown in Eq. B.9.

$$\begin{bmatrix} 2 & 3 & 2 \\ 1 & 4 & 3 \end{bmatrix} \tag{B.9}$$

To transpose it, the rows and columns are switched creating a 2 × 3 matrix. The first row is now the first column. You can see this in the Eq. B.10.

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \\ 2 & 3 \end{bmatrix} \tag{B.10}$$

There are some particular properties of transpositions that you should be familiar with. If you label the first matrix A, then the transposition of that matrix is labeled A^T. Continuing with the original matrix being labeled A, there are a few properties of matrices that need to be described as outlined in Table B.1.

Table B.1 Basic properties of matrix transposition

Property	Explanation
$(A^T)T = A$	If you transpose, the transposition of A, you get back to A
$(cA)^T = cA^T$	The transposition of a constant, c multiplied by an array A, is equal to multiplying the constant c by the transposition of A
$(AB)^T = B^TA^T$	A multiplied by B then the product transposed is equal to B transposed multiplied by A transposed
$(A + B)^T = A^T + B^T$	Adding the matrix A and the matrix B, then transposing the sum is equal to first transposing A and B, then adding those transpositions
$A^T = A$	If a square matrix is equal to its transpose, it is called a symmetric matrix

Table B.1 is not exhaustive, rather, it is a list of some of the most common properties regarding matrices. These properties are not generally particularly difficult to understand. Another operation particular to matrices is finding a submatrix of a given matrix. A submatrix is any portion of a matrix that remains after deleting any number of rows or columns. Consider the 5×5 matrix shown in Eq. B.11

$$\begin{bmatrix} 2 & 2 & 4 & 5 & 3 \\ 3 & 8 & 0 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 4 & 3 & 1 & 2 & 4 \\ 1 & 2 & 2 & 0 & 3 \end{bmatrix} \tag{B.11}$$

If you remove the second column and second row, as shown in Eq. B.12.

B.12

You are left with the matrix shown in Eq. B.13.

$$\begin{bmatrix} 2 & 4 & 5 & 3 \\ 2 & 2 & 2 & 1 \\ 4 & 1 & 2 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix} \tag{B.13}$$

That matrix shown in Eq. B.13 is a sub matrix of the original matrix.

Another item that is important to matrices is the identity matrix. Now this item does have an analog in integers and real numbers. Regarding the addition operation, the identity element is 0. Any number + 0 will still be the original number. In multiplication the identity element is 1. Any number * 1 is still the original number. The identity matrix functions in much the same manner. Multiplying a matrix by its identity matrix, leaves it unchanged. To create an identity matrix, just have all the elements along the main diagonal set to 1, and the rest to zero. Consider the following matrix:

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 0 & 3 \end{bmatrix}$$

Now consider the identity matrix. It must have the same number of columns and rows, with its main diagonal set to all 1s and the rest of the elements all 0s. The identity matrix looks like this:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If you multiple the original matrix by the identity matrix, the product will be the original matrix. You can see this in Eq. B.14.

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 0 & 3 \end{bmatrix} \tag{B.14}$$

Another special type of matrix is a unimodular matrix. Unimodular matrices are also used in some lattice-based algorithms. A unimodular matrix is a square matrix of integers with a determinant of +1 or −1. Recall that a determinant is a value which is computed from the elements of a square matrix. The determinant of a matrix A is denoted by |A|. In the next section we will explore how to calculate the determinant of a matrix.

Determinant of a Matrix

Next, we will turn our attention to another relatively easy computation, the determinant of a matrix. The determinant of a matrix A is denoted by |A|. An example of a determinant in a generic form is as follows:

$$|\mathrm{A}| \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

A more concrete example might help elucidate this concept:

$$|\mathrm{A}| \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = (2)(2) - (3)(1) = 1$$

A determinant is a value which is computed from the individual elements of a square matrix. It provides a single number, also known as a scaler value. Only a square matrix can have a determinant. The calculation for a 2 × 2 matrix is simple enough, we will explore more complex matrices in just a moment. However, what does this single scalar value mean? There are many things one can do with a determinant, most of which we will not use in this text. It can be useful in solving linear equations, changing variables in integrals (yes linear algebra and calculus go hand in hand); however, what is immediately useable for us is that if the determinant is non-zero then the matrix is invertible.

What about a 3 × 3 matrix, such as that shown in Eq. B.15.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \tag{B.15}$$

This calculation is substantially more complex. There are a few methods to do this. We will use one called "expansion by minors." This method depends on breaking the 3 × 3 matrix into 2 × 2 matrices. The 2 × 2 matrix formed by b_2, c_2, b_3, c_3, shown in Eq. B.16, is the first.

B.16

This one was rather simple, as it fits neatly into a contiguous 2 × 2 matrix. But to find the next one, we have a bit different selection as shown in Eq. B.17.

B.17

The next step is to get the lower left corner square matrix as shown in Eq. B.18.

B.18

As with the first one, this one forms a very nice 2 × 2 matrix. Now what shall we do with these 2 × 2 matrices? The formula is actually quite simple and is shown in the Eq. B.19. Note that det is simply shorthand for determinant.

$$\det\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = a_1 \det\begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - a_2 \det\begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + a_3 \det\begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} \tag{B.19}$$

We take the first column, multiplying it by its cofactors, and with a bit of simple addition and subtraction, we arrive at the determinant for a 3 × 3 matrix. A more concrete example might be useful. Let us calculate the determinant for this matrix:

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 0 & 3 \end{bmatrix}$$

This leads to

$$3^* \det\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = 3^* \left(\left(1^* 3\right) - \left(2^* 0\right) \right) = 3(3) = 9$$

$$2^{*}\det\begin{bmatrix}1 & 2\\ 3 & 3\end{bmatrix} = 2^{*}\left(\left(1^{*}3\right)-\left(2^{*}3\right)\right) = 2\left(-3\right) = -6$$

$$1^{*}\det\begin{bmatrix}1 & 1\\ 3 & 0\end{bmatrix} = 1^{*}(\left(\left(1^{*}0\right)-\left(1^{*}3\right)\right) = 1\left(-3\right) = -3$$

And that leads us to $9 - (-6) + (-3) = 12$.

Yes, that might seem a bit cumbersome, but the calculations are not overly difficult. We will end our exploration of determinants at 3×3 matrices. But yes, one can take the determinant of larger square matrices. One can calculate determinants for matrices that are 4×4, 5×5, and as large as you like. However, our goal in this chapter is to give you a bit of general foundation in linear algebra, not to explore every nuance of linear algebra.

Vectors and Vector Spaces

Vectors are an essential part of linear algebra. We normally represent data in the form of vectors. In linear algebra, these vectors are treated like numbers. They can be added and multiplied. A vector will look like what is shown here:

$$\begin{bmatrix}1\\ 3\\ 2\end{bmatrix}$$

This vector has only integers; however, a vector can have rational numbers, real numbers, even complex numbers (which will be discussed later in this chapter). Vectors can also be horizontal, as shown here:

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}$$

Often you will see variables in place of vector numbers, such as

$$\begin{bmatrix}a\\ b\\ c\end{bmatrix}$$

The main point that that you saw in the previous section is that one can do math with these vectors as if they were numbers. You can multiple two vectors together, you can also multiply a vector by a scaler. Scalers are individual numbers, and their name derives from the fact that they change the scale of the vector. Consider the scaler 3 multiplied by the first vector shown in this section:

$$3\begin{bmatrix}1\\3\\2\end{bmatrix}=\begin{bmatrix}3\\9\\6\end{bmatrix}$$

You simply multiply the scaler, by each of the elements in the vector. We will be exploring this and other mathematical permutations in more detail in the next section. But let us address the issue of why it is called a scaler now. We are viewing the data as a vector, another way to view it would be as a graph. Consider the previous vector [1, 3, 2] on a graph as shown in Fig. B.1.

Now what happens when we perform the scalar operation of 3 multiplied by that vector? We literally change the scale of the vector as shown in Fig. B.2.

Figures B.1 and B.2 may appear identical but look closer. In Fig. B.1, the *x* value goes to 9 whereas in Fig. B.2 the *x* value only goes to 3. We have "scaled" the vector. The term scalar is used because it literally changes the scale of the vector. Formally, a *vector space* is a set of vectors which is closed under addition and multiplication by real numbers. Think back to the earlier discussion of abstract algebra with groups, rings, and fields. A vector space is a group. In fact, it is an abelian group. You can do addition of vectors, and the inverse. You also have a second operation scaler multiplication, without the inverse. Note that the first operation, addition is commutative, but the second operation, multiplication is not.

So, what are basis vectors? If you have a set of elements E (i.e., vectors) in some vector space V, the set of vectors E is considered a basis if every vector in the vector space V can be written as a linear combination of the elements of E. Put another

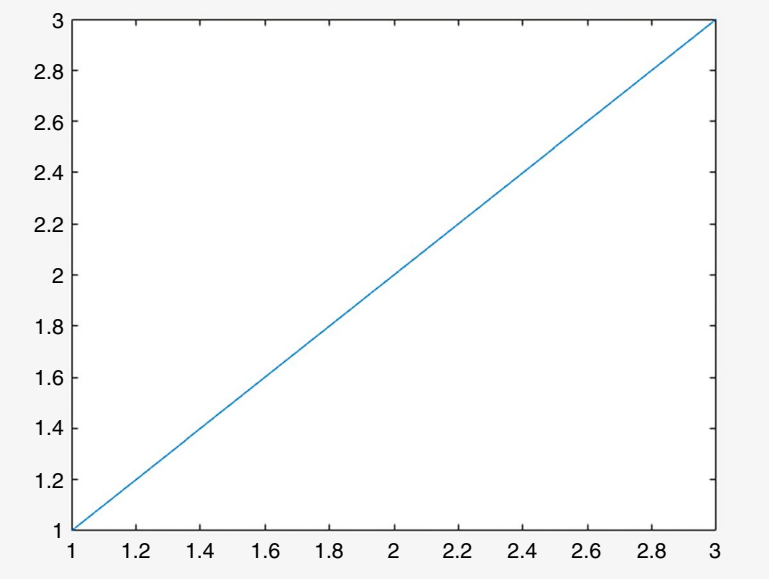


Fig. B.1 Graph of a vector

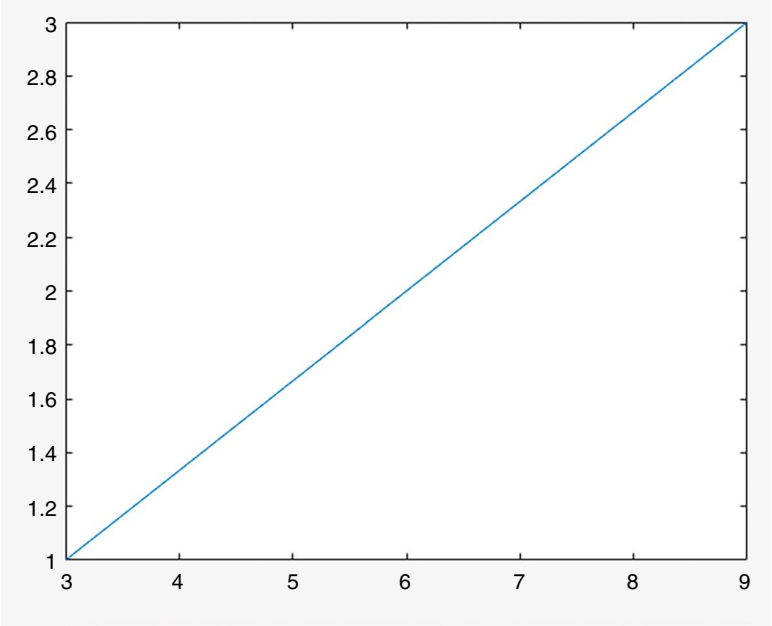


Fig. B.2 Scaling a vector

way, you could begin with the set E, the basis, and through a linear combinations of the vectors in E, create all the vectors in the vector space V. And as the astute reader will have surmised, a vector space can have more than one basis set of vectors.

What is linear dependence and independence? In the theory of vector spaces, a set of vectors is said to be linearly dependent if one of the vectors in the set can be defined as a linear combination of the others; if no vector in the set can be written in this way, then the vectors are said to be linearly independent.

A *subspace* is a subset of a vector space which is a vector space itself, e.g., the plane $z = 0$ is a subspace of R^3 (It is essentially R^2.). We will be looking at R^n and subspaces of R^n.

A set of vectors $A_1, A_2, \ldots, A_n$ is said to span a vector space V if they are in V and every vector in V can be expressed as a linear combination of the $A_1, A_2, \ldots, A_n$.

The largest number of linearly independent vectors in a linear space L is called the dimension of L. A few examples of the vector space of a line and a plane might help elucidate this concept. There is only one linearly independent vector on a line, any two vectors on the line being linearly dependent. There are two linearly independent vectors in the plane, but any three vectors in the plane are linearly dependent. There are three linearly independent vectors in space, but any four vectors are linearly dependent. The set of all vectors lying on a given straight line l forms a linear space, since the sum of two such vectors and the product of such a vector with a real number is again a vector lying on l. The set of all vectors lying in a given plane is also closed with respect to addition and multiplication by real numbers.

Vector Metrics

There are a number of metrics that one can calculate from vector. Each of these is important in some application of linear algebra. In this section we will cover the most common metrics that you will encounter when applying matrix mathematics.

Vector Length

Let us begin this section with a fairly easy topic, the length of a vector, which is computed using the Pythagorean theorem:

$$\text{vector} = \sqrt{x^2 + y^2}$$

Consider the vector:

$$\begin{bmatrix} 2, & 3, & 4 \end{bmatrix}$$

Its length is:

$$\sqrt{2^2 + 3^2 + 4^2} = 5.38$$

This is simple but will be quite important as we move forward. Now we will add just a bit more detail to this concept. The nonnegative length is called the *norm* of the vector. Given a vector v, this is written as ||v||. This will be important later on in this book. One more concept to remember on lengths/norms: if the length is 1, then this is called a *unit vector*.

Dot Product

The dot product of two vectors has numerous applications. This is an operation you will likely encounter with some frequency. The dot product of two vectors is simply the two vectors multiplied. Consider vectors X and Y. Equation B.20 shows what the dot product would be.

$$\sum_{i=1}^{n} X_i Y_i \tag{B.20}$$

Examining a concrete example to see how this works should be helpful. Consider two column vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

The dot product is found by (1*3) + (2*2) + (1*1) = 8.

That is certainly an easy calculation to perform. But what does it mean? Put more frankly, why should you care what the dot product is? Recall that vectors can also be described graphically. You can use the dot product, along with the length of the vectors to find the angle between the two vectors. We already know the dot product is 8. Recall that length:

$$\text{vector} = \sqrt{x^2 + y^2}$$

Thus,

the length of vector X is $\sqrt{1^2 + 2^2 + 1^2} = 2.45$

the length of vector Y is $\sqrt{3^2 + 2^2 + 1^2} = 3.74$

Now we can easily calculate the angle. It turns out that the cos θ = dot product/ length of X*length of Y

or

$$\cos\theta = \frac{8}{(2.45)(3.74)} = .87307$$

Finding the angle from the cosine is straight forward, you probably did this in secondary school trigonometry. But even with just the dot product, we have some information. If the dot product is 0 then the vectors are perpendicular. This is because the cos θ of a 90° angle is 0. The two vectors are referred to as *orthogonal*.

Recall that the length of a vector is also called the vector's *norm*. And if that length/norm is 1, it is the *unit vector*. This leads us to another term we will see frequently later in this book. If two vectors are both orthogonal (i.e., perpendicular to each other) and have unit length (length 1), the vectors are said to be *orthonormal*.

Essentially, the dot product is used to produce a single number, a scaler, from two vertices or two matrices. This is contrasted with the tensor product. In math, a tensor is an object with multiple indices, such as a vertex or array. The tensor product of two vector spaces, V and W, V ⊗ W is also a vector space.

Tensor Product

This is essentially a process where all of the elements from the first vector are multiplied by all of the elements in the second vector. This is shown in Fig. B.3:

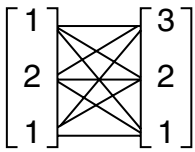


Fig. B.3 Tensor product

Cross Product

This particular metric is very interesting. It illustrates the fact that vector mathematics is inherently geometric. Even if you are working only with vectors, and not seeing the graphical representation, it is there. Let us first describe how you calculate the cross product, then discuss the geometric implications.

If you have two vectors A and B, the cross product is defined as:

$$\mathrm{A} \times \mathrm{B} = |\mathrm{A}|\,|\mathrm{B}|\,\sin(\theta)n$$

|A| denotes the length of A (Note: length is often called magnitude)
|B| denotes the length of B
n is the unit vector that is at right angles to both A and B
θ is the angle between A and B

If you reflect on this a bit, you will probably notice that this requires a three-dimensional space. Using that fact, there is a simple way to calculate the cross product c:

$$cx = \mathrm{A}_y\mathrm{B}_z - \mathrm{A}_z\mathrm{B}_y$$

$$cy = \mathrm{A}_z\mathrm{B}_x - \mathrm{A}_x\mathrm{B}_z$$

$$cz = \mathrm{A}_x\mathrm{B}_y - \mathrm{A}_y\mathrm{B}_x$$

So, consider two vectors: A = [2, 3, 4] and B = [5, 6, 7]

$$\begin{aligned} cx &= 3^*7 - 4^*6 \quad = 21 - 24 = -3 \\ cy &= 4^*5 - 2^*7 \quad = 20 - 14 = 6 \\ cz &= 2^*6 - 3^*5 \quad = 12 - 15 = -3 \end{aligned}$$

The cross product is [−3, 6, −3]

Eigenvalues and Eigenvectors

Eigenvalues are a special set of scalars associated with a linear system of equations (i.e., a matrix equation) that are sometimes also known as characteristic roots, characteristic values proper values, or latent roots. To clarify, consider a column vector we will call v. Then also consider an n x n matrix we will call A. Then consider some scalar λ. If it is true that:

$$\mathrm{A}\mathrm{v} = \lambda\mathrm{v}$$

Then we say that v is an eigenvector of the matrix A and λ is an eigenvalue of the matrix A.

Let us look a bit closer at this. The prefix eigen is actually the German word which can be translated as specific, proper, particular, etc. Put in its most basic form, an eigenvector of some linear transformation T, is a vector that when T is applied to it does not change direction, it only changes scale. It changes scale by the scalar value λ, the eigenvalue. Now we can revisit the former equation just a bit and expand our knowledge of linear algebra:

$$\mathrm{T}(\mathrm{v}) = \lambda v$$

This appears precisely like the former equation, but with one small difference. The matrix A is now replaced with the transformation T. Not only does this tell us about eigenvectors and eigenvalues, but it also tells us a bit more about matrices. A matrix, when applied to a vector, transforms that vector. The matrix itself is an operation on the vector. This is a concept which is fundamental to matrix theory.

Let us add something to this. How do you find the eigenvalues and eigenvectors for a given matrix? Surely it is not just a matter of trial and error with random numbers. Fortunately, there is a very straight forward method, one that is actually quite easy, at least for 2×2 matrices. Consider the following matrix:

$$\begin{bmatrix} 5 & 2 \\ 9 & 2 \end{bmatrix}$$

How do we find its eigenvalues?

Well, the Cayley-Hamilton Theorem provides insight on this issue. That theorem essentially states that a linear operator A is a zero of its characteristic polynomial. For our purposes, it means that

$$\det | \mathrm{A} - \lambda I | = 0$$

We know what a determinant is, we also know that I is the identity matrix. The λ is the eigenvalue we are trying to find. The A is the matrix we are examining. Remember that in linear algebra you can apply a matrix to another matrix or vector, so a matrix is, at least potentially, an operator. So, we can fill in this equation:

$$\det \left| \begin{bmatrix} 5 & 2 \\ 9 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

Now, we just have to do a bit of algebra, beginning with multiplying λ by our identity matrix which will give us:

$$\det \left| \begin{bmatrix} 5 & 2 \\ 9 & 9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

Which in turn leads to

$$\det\left|\begin{bmatrix}5-\lambda & 2\\ 9 & 2-\lambda\end{bmatrix}\right| = 0$$
$$= (5-\lambda)(5-\lambda)-18$$
$$= 10-7\lambda+\lambda^2-18=0$$
$$\lambda^2-7\lambda-8=0$$

This can be factored (note if the result here cannot be factored, things do get a bit more difficult, but that is beyond our scope here):

$$(\lambda-8)(\lambda+1)=0$$

This means we have two eigenvalues:

$$\lambda_1 = 8$$
$$\lambda_2 = -1$$

For a 2 × 2 matrix you will always get two eigenvalues. In fact, for any $n \times n$ matrix you will get n eigenvalues, but they may not be unique.

Now that you have the eigenvalues, how do you calculate the eigenvectors?

We know that:

$$\mathrm{A} = \begin{bmatrix}5 & 2\\ 9 & 2\end{bmatrix}$$
$$\lambda_1 = 8$$
$$\lambda_2 = -1$$

We are seeking unknown vectors, so let us label the vector $\begin{bmatrix}X\\ Y\end{bmatrix}$

Now recall the equation that gives us eigenvectors and eigenvalues

$$\mathrm{Av} = \lambda v$$

Let us take one of our eigenvalues and plug it in:

$$\begin{bmatrix}5 & 2\\ 9 & 2\end{bmatrix}\begin{bmatrix}\mathrm{X}\\ \mathrm{Y}\end{bmatrix} = 8\begin{bmatrix}\mathrm{X}\\ \mathrm{Y}\end{bmatrix}$$
$$\begin{bmatrix}5x+ & 2y\\ 9x+ & 2y\end{bmatrix} = \begin{bmatrix}8\mathrm{X}\\ 8\mathrm{Y}\end{bmatrix}$$

This gives us two equations:

$$5x + 2y = 8x$$

$$9x + 2y = 8y$$

Now we take the first equation and to a bit of algebra to isolate the y value. Subtract the $5x$ from each side to get:

$$2y = 3x$$

Then divide both sides by 2 to get

$$y = 3 / 2\, x$$

It should be easy to see that to solve this with integers (which is what we want) then $x = 2$ and $y = 3$ solve it. Thus, our first eigenvector is

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{with } \lambda_1 = 8$$

You can work out the other eigenvector for the second eigenvalue on your own using this method.

Eigendecomposition

As was mentioned at the beginning of this chapter, Eigendecomposition is key to principle component analysis. The process of Eigendecomposition is the factorization of a matrix into what is called canonical form. That means that the matrix is represented in terms of its eigenvalues and eigenvectors.

Consider a square matrix A that is $n \times n$ with n linearly independent eigenvectors q_i (where $i = 1,\ldots n$). If this is true, then the matrix A can be factorized as shown in Eq. B.21.

$$\mathrm{A} = \mathrm{Q} \wedge \mathrm{A} - 1 \tag{B.21}$$

In Eq. B.21, Q is a square $n \times n$ matrix whose ith column is the eigenvector q_i of A. The symbol $\wedge$ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues. It should be noted that if a matrix A can be eigendecomposed and if none of its eigenvalues are zero, then that matrix is invertible. For more information on Eigendecomposition the following sources may be of use:

https://mathworld.wolfram.com/EigenDecomposition.html
https://personal.utdallas.edu/~herve/Abdi-EVD2007-pretty.pdf

A Little Deeper into Linear Algebra

In this section we will explore some concepts that are often used in quantum physics and quantum computing but may be challenging for some readers. It is recommended that you skip this section until such time as you have mastered the previous linear algebra concepts.

Spread of a Matrix

One concept you will see from time to time is the spread of a matrix. The spread of a matrix is the largest distance in the complex plane between any two eigenvalues of the matrix. For the zero matrix and the identity matrix, the spread is zero. For a projection, the only eigenvalues are zero and one. A projection matrix therefore has a spread that is either 0 if all eigenvalues are equal or 1 if there are two different eigenvalues. You may also see reference to the rank of a matrix. The rank of a matrix A is the dimension of the vector space generated (or spanned) by its columns.

Density Matrix

The density matrix or density operator is an alternate representation of the state of a quantum system for which we have previously used the wavefunction. The density matrix is really an alternative to the wave function for describing the state of a quantum system. The density matrix is defined as the outer product of the wavefunction with its conjugate.

$$\rho(t) = |\psi(t)><\psi(t)|$$

The density matrix is a mathematical object used in quantum mechanics to describe the statistical state of a quantum system. Unlike a state vector which provides a complete description of the state of a quantum system, a density matrix is used when the system is in a mixed state, which is a statistical ensemble of several different possible states. The density matrix ρ for a pure state $|\psi\rangle$ is defined as the outer product $\rho = |\psi\rangle\langle\psi|\rho = |\psi\rangle\langle\psi|$.

This necessitates defining a pure state and a mixed state. A pure state is a state that can be described by a single state vector $|\psi\rangle|\psi\rangle$ in the Hilbert space of the system. A mixed state is a statistical mixture of pure states, each with a certain probability of occurring. It cannot be represented by a single state vector.

In the study of open quantum systems, the density matrix is crucial for understanding how quantum systems interact with their environment through a process called decoherence, which leads to the transition from pure states to mixed states.

Schmidt Decomposition

In addition to the density matrix, the Schmidt decomposition is often used in quantum mechanics, quantum computing, and quantum information theory. The Schmidt decomposition is one of the most important tools for analyzing bipartite pure states in quantum information theory, showing that it is possible to decompose any pure bipartite state as a superposition of coordinated orthonormal states. It is a consequence of the well-known singular value decomposition theorem from linear algebra.

Given a bipartite quantum system described by the Hilbert space $H_A \otimes H_B$, where H_A and H_B are the Hilbert spaces of subsystems A and B respectively, any pure state $|\psi\rangle$ in this combined system can be expressed using the Schmidt decomposition. The Schmidt decomposition theorem states that any pure state $|\psi\rangle$ in $H_A \otimes H_B$ can be written as:

$$1\Psi >= \sum_{i-1}^{r} \lambda_i \mid u_i > \mathrm{A} \otimes 1\mathrm{V}_i > \mathrm{B}$$

where:

$\{|u_i\rangle_A\}$ are orthonormal basis vectors in H_A.
$\{|V_i\rangle_B\}\{$are orthonormal basis vectors in H_B.
λ_i are non-negative real numbers known as Schmidt coefficients.
r is the Schmidt rank, which is the number of non-zero Schmidt coefficients and is at most the minimum of the dimensions of H_A and H_B.

The Schmidt decomposition is used in understanding and quantifying entanglement, which is a key resource in quantum computing and quantum communication.

Complex Numbers

Complex numbers are an extension of the real numbers and are fundamental in various fields of science and engineering, including quantum physics, electrical engineering, and control theory. Complex numbers trace their origin to the simple question of "what is the square root of -1." As a negative multiplied by a negative yields a positive the answer cannot be -1. Rather it is the number i, also called the imaginary unit. A complex number is a number of the form:

$$z = a + bi$$

where:

- a and b are real numbers.
- i is the imaginary unit, defined by the property $i^2 = -1$.

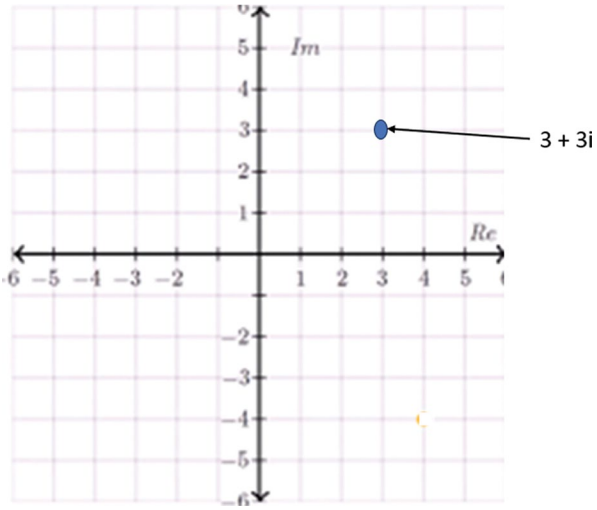


Fig. B.4 The complex plane

Complex numbers can be represented graphically on the complex plane (also known as the Argand plane), where the horizontal axis represents the real part and the vertical axis represents the imaginary part. A complex number $z = a + bi$ is represented as the point (a,b) in this plane. This is shown in Fig. B.4.

The magnitude (or modulus) of a complex number $z = a + bi$ is given by the following formula:

$$|z| = \sqrt{a^2 + b^2}$$

This represents the distance of the point (a,b) from the origin (0,0) in the complex plane.

Basically, the real axis is x, the imaginary x is z and r or hypotenuse is the modulus of the complex number. This can also be expressed as

$$r = |z| = \sqrt{\left[\mathrm{Re}(z)\right]^2 + \left[Im(z)\right]^2} = \sqrt{a^2 + b^2}$$

Mathematically this is often described as the argument of the complex number z. In quantum mechanics and computing, it will often be called the phase. The argument (or phase) of a complex number is the angle θ formed with the positive real axis, given by the following formula:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

A complex number can also be expressed in polar form as:

$$z = r\left(\cos\theta + i\sin\theta\right)$$

where r is the magnitude and θ is the argument. If you know theta (the angle) you can compute the complex number. If you know the complex number, you can compute theta by

$$\theta = \arctan\left(y / x\right) = \tan^{-1}\left(y / x\right)$$

This can also be written using Euler's formula:

$$z = re^{i\theta}$$

Arithmetic Operations

- *Addition*: The sum of two complex numbers $z_1 = a + bi$ and $z_2 = c + di$

$$z_1 + z_2 = \left(a + c\right) + \left(b + d\right)i$$

- *Subtraction*: The difference between two complex numbers z_1z_1 and z_2z_2 is:

$$z_1 - z_2 = \left(a - c\right) + \left(b - d\right)i$$

- *Multiplication*: The product of two complex numbers z_1z_1 and z_2z_2 is:

$$z_1 \cdot z_2 = \left(ac - bd\right) + \left(ad + bc\right)i$$

- *Division*: The quotient of two complex numbers z_1 and z_2 (where $z_2 \neq 0$) is:

$$\frac{z_1}{z_2} = \frac{\left(ac + bd\right) + \left(bc - ad\right)i}{c^2 + d^2}$$

Conjugate

The complex conjugate of a complex number $z = a + bi$ is:

$$\overline{z} = a - bi$$

The conjugate has the same real part but an opposite sign for the imaginary part. The product of a complex number and its conjugate is a real number:

$$z \cdot \overline{z} = a^2 + b^2 = \left|z\right|^2$$

Euler's formula relates complex exponentials to trigonometric functions:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

This formula is fundamental in various applications, particularly in signal processing and quantum mechanics.

Complex numbers play a crucial role in quantum physics, providing the mathematical framework necessary to describe and predict the behavior of quantum systems. The state of a quantum system is described by a wave function, denoted as ψ, which is a complex-valued function. The wave function contains all the information about the system and its evolution over time. The square of the magnitude of the wave function, $|\psi|2|\psi|2$, gives the probability density of finding a particle in a particular position or state. The Schrödinger equation, which is fundamental to nonrelativistic quantum mechanics, describes how the wave function evolves over time. It is a differential equation involving complex numbers:

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\psi$$

Here, i is the imaginary unit, $\hbar$ is the reduced Planck constant, $\hat{H}$ is the Hamiltonian operator (which represents the total energy of the system), and ψ is the wave function. The presence of the imaginary unit ii is essential for the proper description of the time evolution of quantum states.

Quantum mechanics deals with probabilities in a way that is fundamentally different from classical mechanics. The probability of an event is calculated using probability amplitudes, which are complex numbers. The probability of an event is found by taking the square of the magnitude of the corresponding probability amplitude.

Quantum systems exhibit interference and superposition, phenomena that are naturally described using complex numbers. When two or more wave functions overlap, their complex amplitudes add together, leading to constructive or destructive interference. This is a direct consequence of the properties of complex numbers. Another role for complex numbers is in the evolution of a closed quantum system is governed by unitary transformations. These are transformations that preserve the total probability (i.e., the norm of the wave function). Unitary operators, which are essential in quantum mechanics, are defined using complex numbers and ensure that the evolution of quantum states is consistent with the principles of quantum theory.

The description of quantum spin involves complex numbers. The Pauli matrices, which are used to describe the spin-½ particles, are complex matrices that operate on two-component spinors. These matrices are fundamental in the study of spin and the behavior of particles with intrinsic angular momentum

In quantum computing, the state of a quantum bit (qubit) is represented as a vector in a complex Hilbert space. Quantum gates, which manipulate qubits, are represented by unitary matrices with complex entries. The use of complex numbers is essential for the representation and manipulation of quantum information.

Summary

If you wish to go further with linear algebra you may find the following resources useful:

The YouTube channel 3 Blue 1 Brown has an excellent series of videos on linear algebra https://www.youtube.com/watch?v=fNk_zzaMoSs&t=35s

McMahon, D. (2005). Linear algebra demystified. McGraw Hill Professional. This is a good resource for basic linear algebra.

Schneider, Hans; Barker, George Phillip. Matrices and Linear Algebra (Dover Books on Mathematics). Dover Publications

The website An Intuitive Guide to Linear Algebra https://betterexplained.com/articles/linear-algebra-guide/

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C. Easttom, *Hardware for Quantum Computing*,
https://doi.org/10.1007/978-3-031-66477-9